

# An agent-based model to understand the effect of interactions between federal, state, and local governance on the distribution of educational resources

Davoud Taghawi-Nejad

**Abstract.** Income inequality has declined steadily in Brazil, but much less is known about the equality of service provision. To test policies intending to increase equality of service distribution in education, I created an Agent-Based Model (ABM) that simulates the education spending on the national, state and municipal levels. I show that the current policy, to redistribute 80 percent of each municipality's educational budget centrally on the state level, has a positive effect on equality. I find that an increase in centralization increases distributional equality, but with diminishing returns. I provide a graph that shows the correspondence between centralization and equality of service provision.

**Keywords:** Agent-Based Modeling; Brazilian Education System; Equality of Service Provision

## 1 Introduction

Income inequality has declined steadily in Brazil, but much less is known about the equality of service provision.[1] In the field of education, the answers to these problems are not straightforward. In the case of the Brazilian education system, the funding structure is complicated. Interactions between spending requirements, funding sources, and transfers make it hard to measure the distribution of service provision and difficult to predict the impact of policy measures on equality of service provision.

For example, by Article 212 of the constitution, every municipality must spend at least 25 percent of its revenue on education. However, this includes transfers to the state-level redistribution agency, Fundeb. The part of the 25 percent transferred to Fundeb depends on the type of tax that generated the revenue. The transfer from Fundeb back to the municipalities depends on the number and type of students. However these transfers do not have to be spent on the specific type of students that it has been transferred for. There are 26 Fundeb funds, one for each state. The federal government complements a state's Fundeb if the resources are not sufficient to meet the 2280 per-student requirement for all its municipalities. In short, a detailed modeling of the funding structure is necessary to judge whether the system inhibits or fosters inequality.

In Brazil "Ensino Médio" (high school) is the constitutional responsibility of the state, while universities ("Ensino Superior") are the responsibility of the federal government. However, there are secondary schools and universities administered by government levels that are not constitutionally obliged to offer this level of education.

To test policies, I created an Agent-Based Model (ABM) that simulates education spending on the federal, state and municipal levels. The model should have 5,623<sup>1</sup> agents: the Fundeb educational fund, the states and the 26 Brazilian states as well as 5,570 municipalities. Each of the municipal agents is characterized by its educational outcomes. This will allow us to predict the distributional effect of policies and spending on the municipalities. In addition, as I model the effectiveness of education spending, I can make predictions about the distributional effects in educational outcomes.

Of the federal's total tax revenue, 18 percent must be spent on education; the states and municipalities have to spend at least 25 percent of their tax revenue. All agents have decision rules and legal constraints that establish the

---

<sup>1</sup> The accounting data on which this model relies is only available for 4860 municipalities. Municipalities for which no accounting data exists are ignored.

allocation of their taxes. However, the researcher or policymaker can also override the decision rule to make predictions about a policy he or she wants to test.

Based on their tax revenue, the decision rule and legal constraints, all agents spend money on educational programs. The states transfer money to fundeb and spend money on state schools in the municipalities. Municipalities also transfer money to fundeb and spend money on municipal schools. Fundeb, the state fund, redistributes educational resources, back to the municipalities and state schools. The federal state transfers money to fundeb, if the resources are insufficient to attain a minimum per student amount. Agents have decision rules that mimic real-world spending rules and patterns. The constraints include constitutional and legal minimum spending rules. With this ABM I can test the different spending rules, different revenue schemes and a different federal structure, such as centralization or decentralization. This will provide insights about distribution of municipal spending on education; distribution of educational outcomes; evolution of the total educational spending for individual municipalities, states and the federation; and the evolution of educational outcomes for individual municipalities and in aggregate.

The theoretical literature claims that federalism is antithetical to income inequality reduction. [3] However, I argue that a detailed model of the decision and funding structure needs to be built to test this hypothesis. The empirical literature paints a more detailed picture, in which models of federal states vary. For example, the Anglo-Saxon countries have much less homogeneous policies than the Germanic ones. [4] Looking at the educational policy and funding structure in Brazil, it becomes apparent that a detailed modeling of the structure is necessary to judge whether the federal system inhibits or fosters inequality. As Arretche (2015) states in [1], “Instead, a model of interregional income redistribution and sub-national decision-making autonomy is a better predictor of the potential for place-inequality reduction.”

The framework to test policies is an ABM that simulates the education spending on the national, state and municipal levels. This model enables us to predict the distributional effect of policies and spending on the municipalities. In an extension, the effectiveness of education spending will be modeled, thus making it possible to make predictions about the distributional effects in educational outcomes.

An Agent-Based Model (ABM) is a formal modeling technique that allows for less stylized modeling of social, political and economical systems than traditional analytical methods. In an ABM, a set of individual agents are simulated. They are modeled as individual computer programs that interact in an environment. The environment includes a combination of geographic, economic, institutional, and legislative circumstances. ABMs can model any scenario that is verbally expressible, but unlike verbal models, they enforce logical consistency between model and conclusion, which is more akin to analytical models. In addition, they can deliver numerical output of a simulation that can be plotted as graphs and maps.

ABMs can be used for inductive reasoning; an approach called generative social science. [2] Using the generative social science approach enables the researcher to show that a given behavior in a given environment (geographic, economic, institutional and legislative circumstances) leads to a specific outcome.

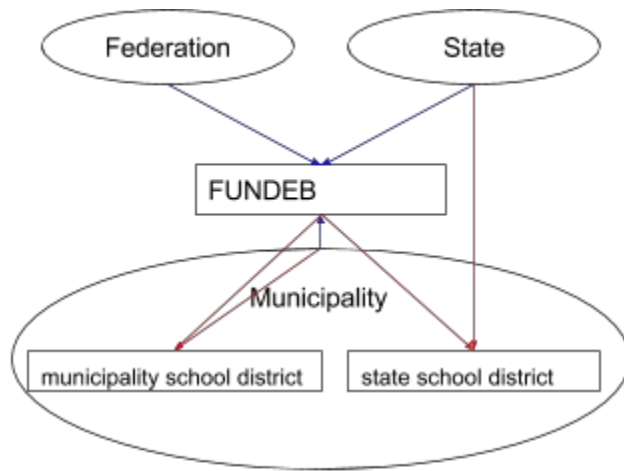
ABMs also can be used for policy simulation. When a model with a realistic environment is created and the behavioral variables are calibrated, the model simulates reality and it can be determined what outcome a specific policy causes. This allows a web-based decision support system to be developed for use by policy makers.

Web-based decision support systems are an effective way of developing policy recommendations in a research context.[5] Policy makers are often characterized as having a short attention span. Therefore, the traditional delivery of academic policy advice—paper and presentation—usually does not convey more than a few bullet points. This model is delivered as a simulation-based web-decision support system (DSS), where policymakers can simulate potential policy options, play with parameters and develop an understanding of how the policy options will

affect the outcomes. Model-based DSS are based on calibrated analytical models, in this case an Agent-Based Model.

## 2 The Model

The simulation simulates the financial flows among the 4860 municipalities, 4860 school districts that are funded directly by the state, the 26 state Fundeb (educational funding agencies), the 26 states and the federal government. Schools can be either municipal, “estadual” (state), federal or private. The 4860 municipal school districts are in charge of the municipal schools, while the 4860 state school districts are in charge of the “estadual” schools in the cities. Municipal and estadual schools exist in the same physical municipality. We don’t model private schools and ignore federal schools, as they are rare.



The 26 Fundeb are the state’s redistribution agencies. They receive a percentage of the tax income from the municipality and the states. If the funds are not sufficient, to guarantee a minimum per student spending, the federation completes their funds. Once the funds are collected FUNDEB redistributes their entire funds to the municipal and state school districts in the municipalities. The Municipality and the State also spend money on their school districts.

### 2.1 The agents in detail

#### The 4860 municipal school district agents and 4860 state school districts

Every municipality in Brazil is represented by two agents: an agent that is in charge of the municipal schools and an agent that is in charge of the state schools in the particular municipality. The municipal school district agents have both a taxation component and a school spending component. The state school districts, on the other hand, have only a spending component, as all their income is coming from state and Fundeb resources.

## **The taxation component of the municipal agents<sup>2</sup>**

Every municipality has the following variables to describe its characteristics:

- the tax base, which is taken from the Brazil's accounting data<sup>3</sup>
- accounting tax base for the transfers to Fundeb, also from the same source<sup>4</sup>
- policy variables regarding taxation: minimum spending percentage and percentage to be transferred to Fundeb

## **Common education finance components of municipal and state school district agents**

Every municipal and state school district has the following variables to describe its characteristics:

- the number of pupils, for each school type in this district<sup>5</sup>

Transactions occur based on these four variables and the interaction with other agents. Each agent has an accounting book, where these transactions are booked; for every account, transfers in and out are recorded. The total inflows, total outflows and net-flows can be retrieved.

There are three kinds of accounting books. First, there are origin accounts, into which the inflows from outside the education system are booked. Two examples are the 25 percent of the tax base of a municipality that is directed to education and a transfer from Fundeb. Second, there are transfer accounts for transfers within the educational system of the municipality. Last, there are destination accounts, where spending on education is booked.

A municipality in the simulation has the following accounts:

- origin - income
- transfer - Education General
- destination - spending

---

<sup>2</sup> Brazil has 5570 municipalities, however only 4860 of them have reported their tax income, educational spending, transfers to the Fundeb educational funds and number of pupils.

<sup>3</sup> The data is retrieved from "Financas do Brasil dados contabeis dos municipios," Year 2012 Volume LVIII, by the "Ministerio da Fazenda, Secretaria do tesouro nacional"

([http://www.tesouro.fazenda.gov.br/pt\\_PT/finbra-financas-municipais](http://www.tesouro.fazenda.gov.br/pt_PT/finbra-financas-municipais))

<sup>4</sup> The transfers to Fundeb are not calculated from the whole tax base, but from a subset I call the accounting tax base, which is calculated as follows:

- Fundo de Participação dos Municípios – FPM
- Imposto sobre Circulação de Mercadorias e Serviços – ICMS
- Imposto sobre Produtos Industrializados, proporcional às exportações – IPIexp
- Desoneração de Exportações (LC 87/96)
- Imposto sobre Transmissão Causa Mortis e Doações – ITCMD
- Imposto sobre a Propriedade de Veículos Automotores – IPVA
- Quota Parte de 50 percent do Imposto Territorial Rural devida aos Municípios – ITRm

<sup>5</sup> The data is retrieved from "Censo Escolar," Year 2012, by the "Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira" (<http://portal.inep.gov.br/basica-censo>)

Every municipal agent executes the following procedure:

1. Transfer 25 percent of the tax base to the municipality's "Education General" account<sup>6</sup>
2. Transfer 20 percent of the accounting tax base from its own "Education General" account to Fundeb's "Education General" account
3. (receive transfers and pool them in the "Education General" account)
4. Spend the money from the Education General account

All rules can be quantitatively and, with more effort, qualitatively changed. For example, it is possible to run a simulation where all educational spending is centralized.

### **State**

Each of the 26 states has the same variables and procedures as a municipality, with two exceptions. First, the taxes that are used to calculate the minimum spending and the transfer to Fundeb are different. Second, states do not fund schools directly, but instead transfer the money to state school districts, which in turn spend the money.

### **Fundeb**

There are 26 Fundeb agents. The Fundeb agents receive taxes and transfer funds to the school districts, which spend them on education. The procedure is for Fundeb to divide its total resources by the number of pupils. Depending on the kind of school, the per-pupil spending is adjusted according to the table below. The money is then transferred to the school district proportionally to its 'adjusted' student number.

Like the municipal agents, Fundeb agents have accounting books.

The Fundeb agents in the simulation have the following accounts:

- transfer - Creche
- transfer - Séries iniciais do ensino fundamental
- transfer - Educação de jovens e adultos com avaliação no processo
- transfer - Ensino médio integrado à educação profissional
- transfer - Séries iniciais do ensino fundamental
- transfer - Ensino médio
- transfer - Séries finais do ensino fundamental
- transfer - Pré-escola

Every Fundeb agent executes the following procedure:

1. (receive funds from municipalities and states)
2. Transfer funds to the "General Account" of each municipality school district and state district

---

<sup>6</sup> In a line of double entry accounting it gets booked as income in "Origin - Income" and as an asset in "Transfer - Education General"

Etapas e modalidades e segmentos	Resolução Nº 01, de 15/02/2007	Portaria Nº 41, de 27/12/2007	Portaria Nº 932, de 30/07/2008	Portaria Nº 777, de 10/08/2009	Portaria Nº 873, de 01/07/2010	Portaria Nº 1.322, de 21/09/11	Resolução Nº 8, de 25/07/2012	Resolução Nº 01, de 31/12/2013	Resolução Nº 01, de 24/07/2014
	2007	2008	2009	2010	2011	2012	2013	2014	2015
1. Creche	0,80	-	-	-	-	-	-	-	-
2. Creche pública de tempo parcial	-	0,80	0,80	0,80	0,80	0,80	0,80	1,00	1,00
3. Creche conveniada de tempo parcial	-	0,80	0,80	0,80	0,80	0,80	0,80	0,80	0,80
4. Creche pública de tempo integral	-	1,10	1,10	1,10	1,20	1,30	1,30	1,30	1,30
5. Creche conveniada de tempo integral	-	0,85	0,95	1,10	1,10	1,10	1,10	1,10	1,10
6. Pré-escola	0,90	-	-	-	-	-	-	-	-
7. Pré-escola parcial	-	0,90	1,00	1,00	1,00	1,00	1,00	1,00	1,00
8. Pré-escola integral	-	1,15	1,20	1,25	1,30	1,30	1,30	1,30	1,30
9. Anos iniciais – ensino fundamental urbano	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
10. Anos iniciais – ensino fundamental rural	1,05	1,05	1,05	1,15	1,15	1,15	1,15	1,15	-
11. Anos iniciais – ensino fundamental no campo	-	-	-	-	-	-	-	1,15	1,15
12. Anos finais – ensino fundamental urbano	1,10	1,10	1,10	1,10	1,10	1,10	1,10	1,10	1,10
13. Anos finais – ensino fundamental rural	1,15	1,15	1,15	1,20	1,20	1,20	1,20	1,20	-
14. Anos finais – ensino fundamental campo	-	-	-	-	-	-	-	1,20	1,20
15. Ensino fundamental integral	1,25	1,25	1,25	1,25	1,30	1,30	1,30	1,30	1,30
16. Ensino médio urbano	1,20	1,20	1,20	1,20	1,20	1,20	1,20	1,25	1,25
17. Ensino médio no campo	1,25	1,25	1,25	1,25	1,25	1,30	1,30	1,30	1,30
18. Ensino médio integral	1,30	1,30	1,30	1,30	1,30	1,30	1,30	1,30	1,30
19. Ensino médio integrado à educação profissional	1,30	1,30	1,30	1,30	1,30	1,30	1,30	1,30	1,30
20. Educação especial	1,20	1,20	1,20	1,20	1,20	1,20	1,20	1,20	1,20
21. Educação indígena e quilombola	1,20	1,20	1,20	1,20	1,20	1,20	1,20	1,20	1,20
22. Educação de jovens e adultos com avaliação no processo	0,70	0,70	0,80	0,80	0,80	0,80	0,80	0,80	0,80
23. Educação de jovens e adultos integrada à educação profissional de nível médio, com avaliação no processo	0,70	0,70	1,00	1,00	1,20	1,20	1,20	1,20	1,20

## The Federal Government

The federal agent executes the following rules:

1. Transfer 18 percent of the tax base to “Transfers General”
2. Transfer funds to Fundeb's in cases where the per-pupil minimum spending requirement is not met
3. Spend remaining funds on university education

## 3 Applications of the Model

This model is able to replicate the spending and outcome patterns observed in Brazil. In an extension, educational outcomes will also be predicted. Alternative policies or scenarios of educational funding can also be tested. For example, one could test the following:

- different spending rules
- different revenue schemes
- a different federal structure, such as centralization or decentralization
- different spending constraints

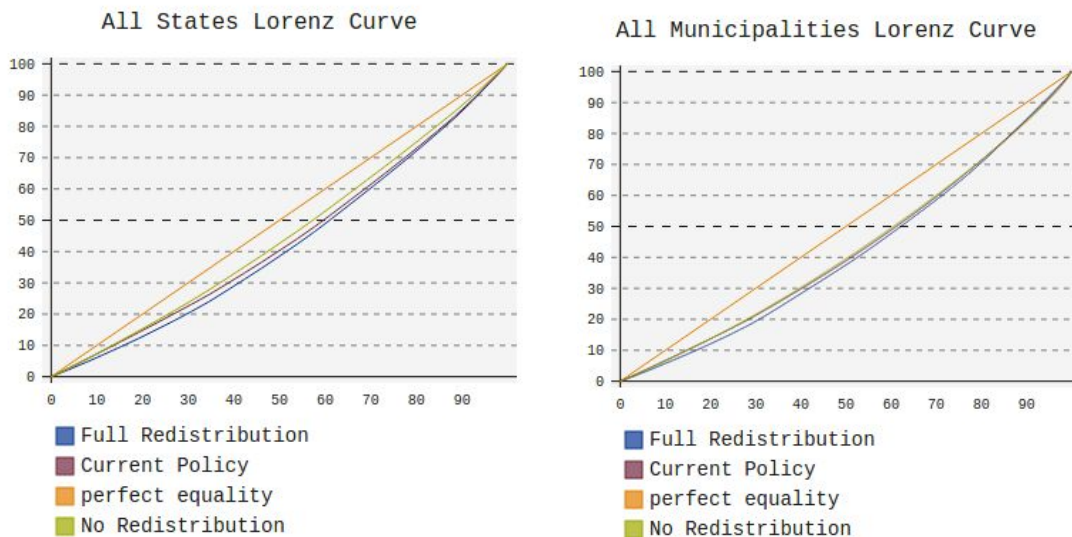
- different transfer rules

By simulating these policies and scenarios, the model could be used to investigate the following:

- distribution of service provision
- evolution of educational outcomes, both for individual municipalities and in aggregate

For example, I could investigate the distributional outcome if the states do not spend on municipalities and all spending comes from the federal government. Further, I could test what would happen if no redistribution occurred or if all spending was centralized.

#### 4.4 Preliminary Analysis



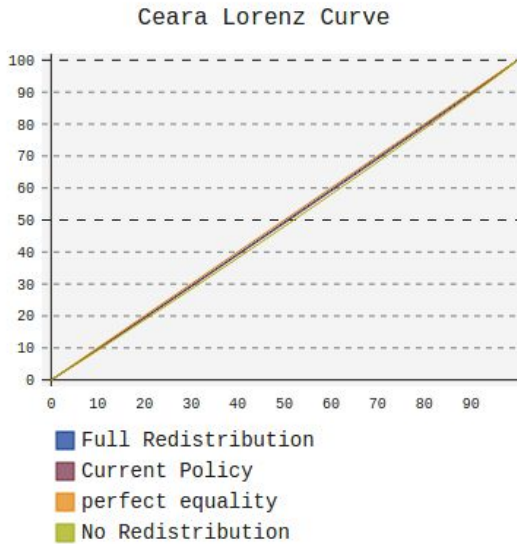
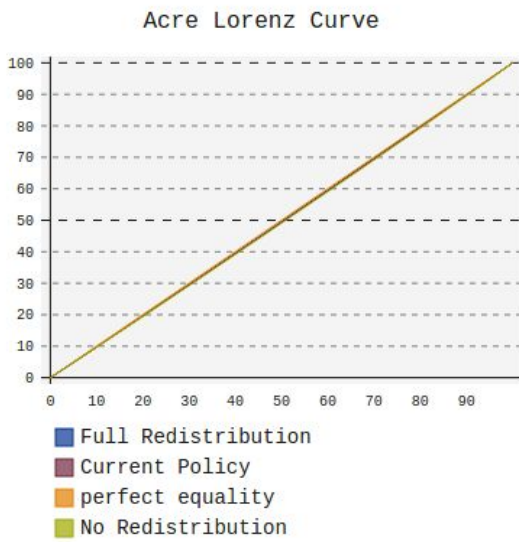
As a primary analysis, we run the simulation with the current policy, full redistribution, and no redistribution. We graph the result as Lorenz curves in which the area between a curve and the diagonal line (in yellow) represents the inequality; a larger area represents more inequality.<sup>7</sup>

The first observation is that on the states' Lorenz curve with full redistribution, the current policy and no redistribution are close to each other. The Gini coefficients are 15.58, 12.90, 10.07. Thus, on a state level, the inequality decreased and could decrease more with full redistribution. However a significant inequality, which could only be alleviated by redistribution on national level, remains.

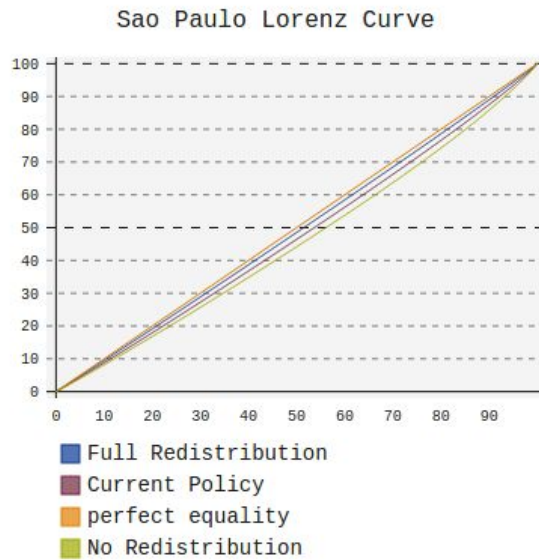
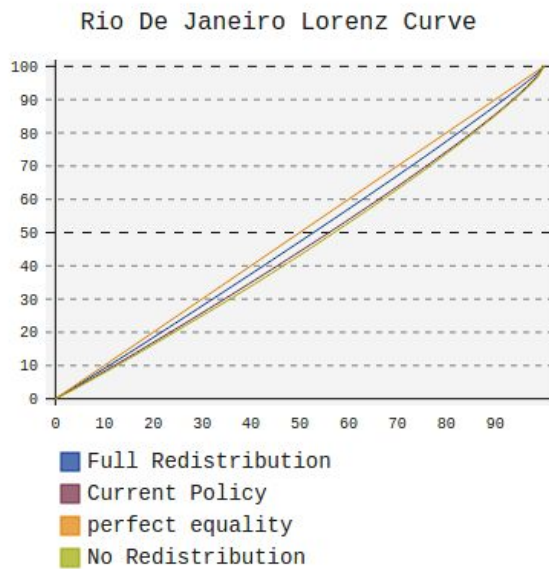
For all municipalities the gini coefficient with full redistribution is 17.31, under the current policy it is 15.36, with no redistribution it would be 15.01. The Lorenz curves of the current policy and no redistribution consequently almost coincide. Particularly for wealthier municipalities, on the upper right part of the curve. The current policy from this angle does not alleviate much inequality and a full redistribution on state level funds could perform significantly better.

The difference between the between-states Gini and the between-municipalities Gini coefficients indicates that a state level analysis is necessary. I picked six of the 26 states that capture all patterns.

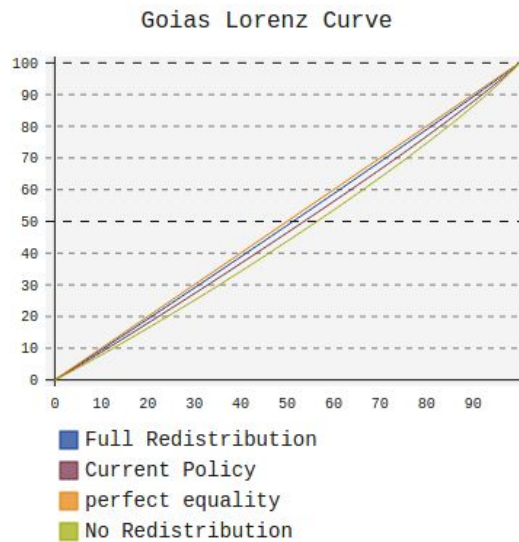
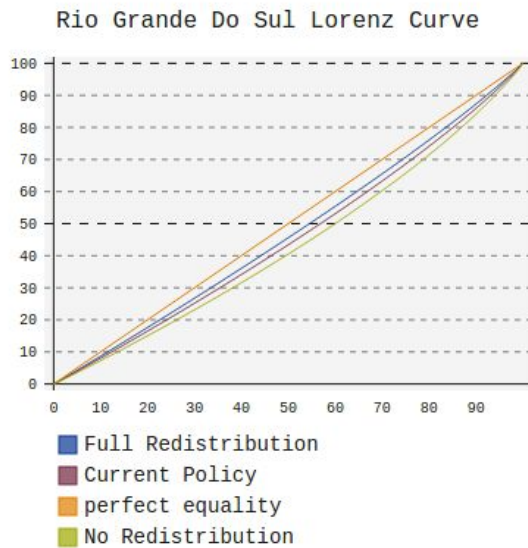
<sup>7</sup> See [https://en.wikipedia.org/wiki/Lorenz\\_curve](https://en.wikipedia.org/wiki/Lorenz_curve) and [https://en.wikipedia.org/wiki/Gini\\_coefficient](https://en.wikipedia.org/wiki/Gini_coefficient)



The state of Acre had a very low inequality to begin with. While the current policy decreased inequality from 1.34 to 0.85, a full redistributive policy would result only in a Gini of 0.78. Ceara, which started with a higher inequality of 2.84, reduced it to 1.53 and could decrease it further to 0.93.







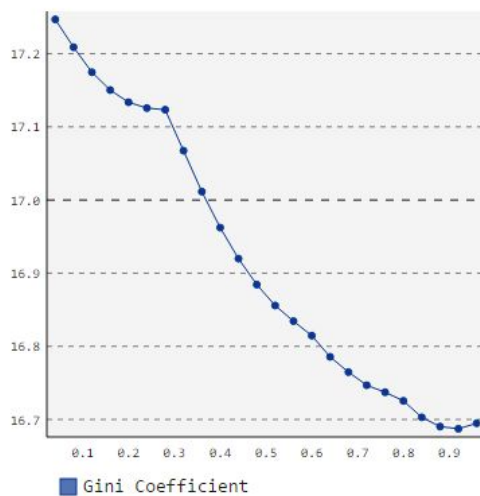
States that began with a higher inequality paint a different picture. In those states, the current policy has a high impact, e.g., in Rio Grande Do Sul, where the Gini decreases from 13.54 to 9.42. In addition, a further decrease to 6.32 could be achieved. In the case of Goias and similar states, the current policy has a high impact. It results in a decrease from 9.04 to 5.25, and it could almost eradicate inequality, with a final Gini of 1.84.

Full redistribution does not achieve full equality because different students types get different transfers. School districts get between 10 and 15 percent more 'ensino fundamental' for students in rural schools than they get for students in urban schools.

State	Full Redistribution	Current Policy	No Redistribution
All Municipalities	17.31	15.36	15.01
All States	15.58	12.9	10.07
Acre	0.78	0.85	1.34
Alagoas	1.15	1.98	2.86
Amapa	1.05	3.9	3.97
Amazonas	1.21	2.23	3.13
Bahia	1.11	2.56	4.18
Ceara	0.93	1.53	2.84
Espirito Santo	1	2.35	4.15
Goias	1.84	5.25	9.04
Maranhao	1.09	2.26	3.26
Mato Grosso	3.42	5.84	7.57
Mato Grosso Do Sul	0.67	3.3	5.66
Minas Gerais	0.8	3.55	7.37
Para	0.73	2.97	4.07
Paraiba	1.23	2.78	6.01
Parana	2.03	4.08	6.81

Pernambuco	0.89	2.28	3.23
Piaui	1.88	3.16	5.13
Rio De Janeiro	4.12	8.64	10.07
Rio Grande Do Norte	1.04	2.88	5.5
Rio Grande Do Sul	6.32	9.42	13.54
Rondonia	0.92	2.1	4.63
Roraima	0.5	0.9	1.58
Santa Catarina	0.78	3.27	8.12
Sao Paulo	2.21	5.35	8.72
Sergipe	0.85	2.48	4.53
Tocantins	0.85	2.37	5.51

Table of Gini coefficients under full, current, and no redistribution



The graph shows the Gini coefficient as a function of the percentages of municipal and state income that are redistributed via Fundeb.

## 4 Conclusion

Educational equality is not yet achieved in Brazil. Redistribution agencies are in place, but it is not simply a question of fine-tuning the redistribution. Given the vast differences in GDP among the Brazilian states, no quantitative change in policy can achieve full equality of service provision: it could only be achieved by redistribution on the federal level.

## 5 Extension

In a future version of this model I want to model not only the financial streams, but also the effect of income on outcomes. For this I will use previous research that estimates educational outcomes for all Brazilian municipalities, and connect it to spending.

Municipal agents are special in the sense that they carry the outcome variables. In reality, each municipality has a population with certain educational outcomes. Spending on education can change the outcomes. In this simulation,

the educational outcomes are represented directly, omitting the modeling of the population. In using this approach, I mimic the Municipal Education Index.<sup>8</sup> Using panel data estimation, I estimate the effect of spending on educational ( $E$ ) outcomes from our empirical data as follows:

$$E_{m,t} = \alpha + \beta f(\text{spending}_{m,t}) + \gamma g(E_{m,t-1}) + \delta h(\text{spending}_{m,t} \cdot E_{i,m,t-1}) + \zeta \text{municipal\_modifier}_m + \varepsilon_{m,t}$$

where  $t$  is time,  $m$  is municipality and  $E$  is measured by the Municipal Education Index. The *municipal\_modifier* variable is a fixed effect that partly depends on the number of employees and the structure of the municipality.

For this extension, the exact functional form will be established during the course of the research. It is possible that spending on different levels will need to be estimated with different coefficients. The outcomes could be further disaggregated, yielding a multitude of equations for individual components of the synthetic index. It might also be necessary to model the areas of spending in detail. This would leave us with a system of simultaneous equations  $E_{i,m,t}$  depending on a variety of coefficients for spending on different areas  $\text{spending}_{j,m,t}$  and their cross-products. The variable  $i$  is a component of the Municipal Education index,<sup>9</sup> and  $j$  is the area of spending.

## Appendix - Data and Parameters

The model's data basis and parameters are derived from the following data sources, tables, laws and decrees:

### The financial data

The data on the 2012 municipal tax income is retrieved from "FINBRA – Finanças do Brasil – Dados Contábeis dos Municípios" ("Finbra - Brazilian Finances - Accounting data of the municipalities").

The data on state tax income is retrieved from "Execução Orçamentária de Estados" ("Final State Budget").

National accounting data is retrieved from "Balanço do Setor Público Nacional – BSPN" ("National Public Sector Accounts").

All financial data can be downloaded at <http://www.tesouro.fazenda.gov.br/contas-anuais>. As it is official tax data, it is the most reliable source, but data for several hundred municipalities is missing. The most recent available year for the municipal data is 2012.

### Data on student numbers

Every year micro data on Brazilian students is collected. This "censo escolar" is the same data that is used to distribute the Fundeb resources. The micro data had to be aggregated on a municipal level. It can be downloaded at <http://portal.inep.gov.br/basica-censo>.

### Legislation

There are two main sources for relevant laws: Article 212 of the constitution, which established minimum spending requirements, and "DECRETO Nº 6.253, DE 13 DE NOVEMBRO DE 2007. - Dispõe sobre o Fundo de Manutenção e Desenvolvimento da Educação Básica e de Valorização dos Profissionais da Educação - FUNDEB, regulamenta a Lei no 11.494, de 20 de junho de 2007, e dá outras providências." The parameters for the model are

---

<sup>8</sup> <http://www.flfch.usp.br/centrodametropole/en/1160> and Arretche (2015)

<sup>9</sup> For example, "Disapproval rate until Xth grade," "Dropout rate until Xth grade," "Age-grade distortion until Xth grade."

established in these two sources, While the weighting factors for student types comes from the “Confederacao Nacional de Municipios - Fatores de Ponderação do FUNDEB 2007-2015,” available at

[http://www.cnm.org.br/portal/images/stories/Links/25072014\\_ponderaes\\_Fundeb\\_2007-2015.pdf](http://www.cnm.org.br/portal/images/stories/Links/25072014_ponderaes_Fundeb_2007-2015.pdf).

## References

1. Arretche, M.: Federalism, Social Policy, and Reductions in Territorial Inequality in Contemporary Brazil. In: Schneider, B.R. (forthcoming): *Dilemmas of Democracy and Development in Brazil: Comparative and Historical Perspectives* (2015).
2. Epstein, J.M.: Agent-Based Computational Models And Generative Social Science. *Complexity* 4(5), 41–60 (1999).
3. Wildavsky, A.: Federalism Means Inequality: Geometry, Political Sociology, and Political Culture. In Golembiewski, R.T., Wildavsky, A. (eds.): *The Costs of Federalism*. Transaction Books, New Brunswick/London, 55-69 (1984).
4. Braun, D. : *Public Policy and Federalism*. Ashgate (2000).
5. Ruppert, T., et al.: Visual Decision Support for Policy Making: Advancing Policy Analysis with Visualization. *Policy Practice and Digital Science*. Springer International Publishing, 321-353 (2015).