Networked Firm Competition in Evolving Product Attribute Spaces

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Abstract. Traditionally, firm competition has been studied in contexts where the product attribute space is assumed to be fixed. Thus, firms deploy their strategies constrained by such a space. However, it is also true that firms exert influence on the product attribute space by offering products with new attributes and variants. As a result, the product attribute space endogenously changes as a consequence of firms' actions. This works touches upon the *co-evolution of the firm strategy and the product space*. Through a networked Cournot competition framework, we develop a computational model in which firms invest in new product variants that are characterized by minimum differentiation with existing ones. That is, firms try to differentiate from other variants, so a new market niche is created, but as minimum as possible so that demand from closely similar existing variants can be "stolen". We study the effects of the evolving space on firm performance and how this dynamics affects and is affected by firms' adaptive behavior.

Keywords: Cournot competition, product attribute spaces, dimensionality, agent-based simulation

1 Background

Attribute spaces of positive integer dimensionality display how demand for, and supply of, organizational services distribute over a number of attributes (dimensions). The space can be a commodity space spun by product characteristics prospective customers evaluate (Lancaster, 1966; Kim et al., 2007; Lacourbe et al., 2009; Adner et al., 2014) or a political issue space within which political representation is offered (Downs, 1957). In organization science, it can also be Blau-space spun by

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adfa, p. 1, 2011.

people's socio-demographic characteristics that motivate their choices with respect to organizational offerings (Boone et al., 2002; Hannan et al., 2007; McPherson, 1983, 2004). Economic and organizational research have already disclosed a number of mechanisms regarding how changing integer dimensionality impacts upon market outcomes (Irmen and Thisse, 1998; Péli and Nooteboom, 1999; Péli and Witteloostuijn, 2009).

A persisting problem of multidimensional space representations is, however, that real-world markets stay normally far from offering, or even recognizing, all analytically possible product feature combinations. Demand distributions over multidimensional attribute spaces normally show a patchy pattern (cf. Bruggeman and Péli, 2014). This problem is even more salient when we consider the situation where firms affect the shape of the product attribute space through price / quantity competition and product differentiation. Therefore, resorting to a Cournot competition framework, we attempt to understand the co-evolutionary implications of firms' strategies and the product attribute space where firms are embedded in.

Dimensionality of attribute spaces is usually accounted for by the number of product attributes relevant for potential customers (Lancaster, 1966). Given the fact that we consider scenarios where not all possible attribute value configurations are available at the same time, we have to take into account the non-integer nature of the product attribute space dimensionality in our model. For such a purpose, we employ a metric that we label "fraction dimension". Our paper demonstrates that fraction dimensionality, defined by the similarity dimension concept of Mandelbrot (1983), is a useful modeling tool of product positioning in multidimensional markets. Using fraction dimension as independent variable has already been shown contributing to explain the fates of large and small scale firms over patchy demand landscapes (García-Díaz, 2008, García-Díaz et al., 2008). As markets mature, increasing crowding makes firms resorting to horizontal product differentiation (Eaton and Lipsey, 1989). Accordingly, fraction dimension increases as proliferating demand patches make the spotty scenery gradually saturated.

Our model reflects the fact that firms are constrained by the current product space structure, but firms also shape such a space. Traditional models in the literature (cf. Irmen and Thisse, 1998) consider the product space as given, but do not take into account the effect of newly introduced products in shaping consumer preferences (Cojocaru et al., 2013).

Thus, we define a "product variant" as a product that specifically targets a given attribute combination, which implies that each point in the product space corresponds to a product variant. Competition in our model resembles what other works label as "networked Cournot competition" (Bimpikis et al., 2014): active product variants have interdependent demands, so the fact that a firm opens up a new variant implies that its product variant demand depends on how many consumers from the existing demand in other variants switch toward the new variant.

Our work also relates to the existing literature in a number of ways: some works have emphasized the role of profit landscapes in shaping firm strategies (see for instance, Robertson and Caldart, 2009). Other works consider the role of firm-level adaptive behavior in Cournot competition across such landscapes (Lenox et al., 2006, 2007) and the effect of strategic positioning in multiattribute spaces (Adner et al., 2014). Nonetheless, all these works consider spaces that are exogenously defined. Here, we contemplate the possibility of having firms reacting upon the existing space, as well as their strategic capabilities to modify it. In our model, firms develop new product variants according to two criteria: (i) firms try to product differentiate, so that there are no competitors in the new product variant, and (ii) new variants have to be as close as possible to existing variants in the attribute space in order to "steal" consumers from the latter. Putting it simple, firms' new variants try to differentiate as minimum as possible from the existing variants. Thus, the attribute space resembles an evolving network of product variants. The model is explained in the next section.

2 The model

2.1 Fundamentals

We conceive a market with an evolving number of product variants with a constant demand of M consumers. Each dimension in the space represents a product attribute. Each variant has a given market demand and is represented as a cell (spot) in a (multi-dimensional) product attribute space. Product variants emerge as a consequence of firm actions in the attribute space. Variants may appear in the market due to exogenous factors (Dawid et al., 2001), but here firms can strategically choose the spatial location of the product variant they want to invest in. Choosing of a new product variant is dependent on firm saturation of existing variants, so firms attempt to soften competition in highly crowded variants by opening up a new spot in the space. Variants are interdependent in the sense that demand from one variant can be dragged into another variant created by a firm. See Figures 1 and 2.

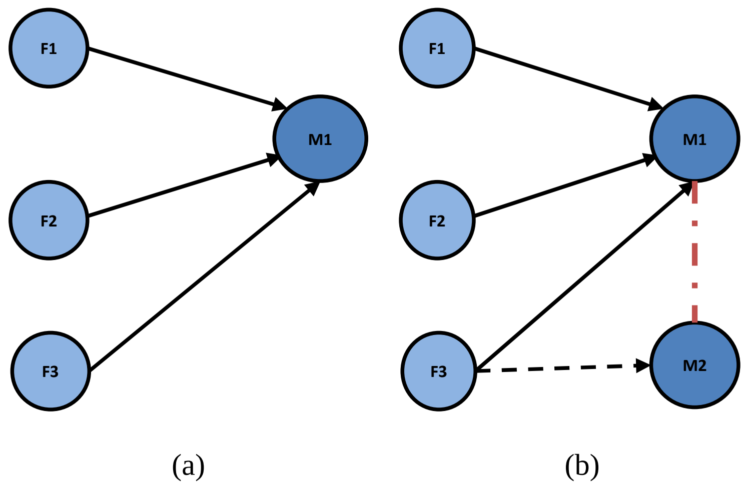

Fig. 1. A market as an evolving bipartite network. F1, F2 and F3 represent firms, while M1 and M2 represent product variants. Panel (a) depicts the situation where all firms compete by the same product variant, while panel (b) illustrates the effect generated by firm F3 in opening a new variant M2. Part of M1 consumers are dragged into variant M2.

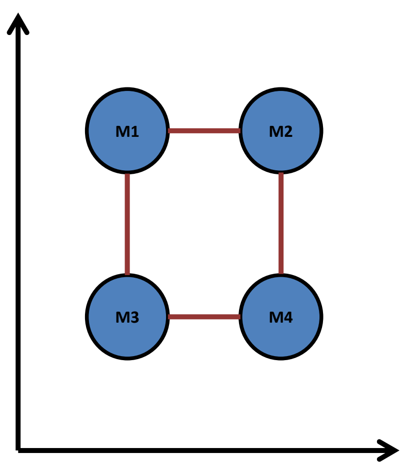

Fig. 2. Variants in attribute space.

2.2 Firm behavior and costs

We have a constant population F of firms which can be of two types: Firms with scale economies (α-type) and firms without them (β-type). At the beginning of the simulation each firm has an equal probability of being either type, but once the type has been chosen it remains constant throughout the simulation. At each product variant j at time t, we assume there is a number of firms nα,j,t with scale economies, and nβ,j,t firms without. We also assume that β-type firms have a constant unit production cost $c_{\beta,j}$ at variant j while α-type firms are assumed to be able to reduce unit production costs as production increases. For an α-type firm we assume that its realized unit production cost at variant j at time t, $c_{\alpha,j,t}$, depends on its production level in this variant. That is, if an α-type firm operating a variant j, say firm i, at time t produces $q_{i,j,t}$, its unit production cost is computed as:

$$c_{\alpha,j,t} = c_{\beta,j} - e_j q_{i,j,t}\,. \tag{1}$$

We assume that each variant has a unique associated cost of $c_{\beta,j}$, which is determined by a uniform probability distribution. The term ej > 0 represents variant j's unit cost reduction impact per unit of production quantity. In order to avoid the possibility of having negative values for $c_{\alpha,j,t}$, we might establish a minimum unit cost c_o, so that $c_{\beta,j} - e_j M \ \geq c_o$, from which we obtain $c_{\beta,j} \geq c_o + e_j M$. We opt for setting $c_o = 0$. At a given time t, firm i calculates its profits at product variant j as follows (hereafter we suppress the time-related index for the sake of clarity in the explanations):

$$\pi_{i,j} = P_j q_i - c_{k,j} q_{i,j}\,. \tag{2}$$

The coefficient $c_{k,j}$ represents the unit production cost, which depends on whether the firms is α-type or β-type, $k = \{\alpha, \beta\}$. Assuming nj firms (of both types) operating at variant j, the product variant j has the following price function:

$$P_j = a - b_j Q_j = a - b_j \sum_{i=1}^{n_j} q_{i,j}\,, \tag{3}$$

where Q_j is total quantity at variant *j*, a and b_j $(a, b_j \geq 0)$ are the price equation intercept and slope values. In each variant, firms get involved in direct competition. Since the price is set by the market, firms choose quantities in order to maximize their profits (what is known as *Cournot* competition). Replacing Eq. (3) in Eq. (2), a given firm *i* that takes advantage of scale economies (i.e., *α-type*) maximizes its benefits at variant *j* according to:

$$\frac{\partial \pi_{i,j}}{\partial q_{i,j}} = a - b_j \sum_{z=1, z\neq i}^{n_j} q_{z,j} - 2b_j q_{i,j} - c_{\beta,j} + 2e_j q_{i,j} = 0, \ \ i = 1,2, \dots n_{\alpha,j}, \tag{4}$$

$$\frac{\partial \pi_{i,j}}{\partial q_{i,j}} = a - b_j Q_j + q_{i,j}\big(2e_j - b_j\big) - c_{\beta,j} = 0, \ \ i = 1,2, \dots n_{\alpha,j}. \tag{5}$$

Summing up over all $n_{\alpha,j}$ firms, we get:

$$n_{\alpha,j}a - n_{\alpha,j}b_jQ_j + (2e_j - b_j)\sum_{i=1}^{n_{\alpha,j}} q_{i,j} - n_{\alpha,j}c_{\beta,j} = 0. \tag{6}$$

In an analogous way, for a *β-type* firm we obtain:

$$\frac{\partial \pi_{i,j}}{\partial q_{i,j}} = a - b_j\sum_{z=1,z\neq i}^{n_j} q_{z,j} - 2b_jq_{i,j} - c_{\beta,j} = 0,\;\; i = n_{\alpha,j}+1, n_{\alpha,j}+2, \dots, n_{\alpha,j} + n_{\beta,j}\;\;, \tag{7}$$

thus, summing up over all $n_{\beta,j}$ firms, we get:

$$n_{\beta,j}a - n_{\beta,j}b_jQ_j - b_j\sum_{i=n_{\alpha,j}+1}^{n_{\alpha,j}+n_{\beta,j}} q_{i,j} - n_{\beta,j}c_{\beta,j} = 0. \tag{8}$$

Knowing that the total number of firms in variant j is $n_j = n_{\alpha,j} + n_{\beta,j}$, and that $Q_j = \sum_{i=1}^{n_{\alpha,j}} q_{i,j} + \sum_{i=n_{\alpha,j}+1}^{n_{\alpha,j}+n_{\beta,j}} q_{i,j}$, we proceed to multiply equation (8) by $-(2e_j - b_j)/b_j$ and add it to equation (6). Then we get:

$$\left[n_{\alpha,j} - [\frac{(2e_j-b_j)}{b_j}]n_{\beta,j}\right]a + \left[n_{\beta,j}(2e_j - b_j) - b_jn_{\alpha,j}\right]Q_j + (2e_j - b_j)Q_j - \left[n_{\alpha,j} - [\frac{(2e_j-b_j)}{b_j}]n_{\beta,j}\right]c_{\beta,j} = 0\,, \tag{9}$$

which yields a total quantity at variant *j* of:

$$Q_j = \frac{n_{\alpha,j} - [\frac{(2e_j-b_j)}{b_j}]n_{\beta,j}}{b_jn_{\alpha,j} - (2e_j-b_j)(n_{\beta,j}+1)}(a - c_{\beta,j})\;\;. \tag{10}$$

Going back to equations (6) and (8) and defining $Q_{\alpha,j} = \sum_{i=1}^{n_{\alpha,j}} q_{i,j}$, $Q_{\beta,j} = \sum_{i=n_{\alpha,j}+1}^{n_{\alpha,j}+n_{\beta,j}} q_{i,j}$, we get:

$$Q_{\alpha,j} = \frac{n_{\alpha,j}(a - b_jQ_{\beta,j} - c_{\beta,j})}{b_jn_{\alpha,j} - (2e_j - b_j)}\,, \tag{11}$$

$$Q_{\beta,j} = \frac{n_{\beta,j}(a - b_jQ_{\alpha,j} - c_{\beta,j})}{b_jn_{\beta,j} + b_j}\,. \tag{12}$$

Solving by substitution equations (11) and (12), we obtain:

$$Q_{\alpha,j} = \frac{n_{\alpha,j}}{b_j(n_{\alpha,j}+n_{\beta,j}+1) - 2e_j(n_{\beta,j}+1)}(a - c_{\beta,j})\,, \tag{13}$$

$$Q_{\beta,j} = \frac{[\frac{(b_j-2e_j)}{b_j}]n_{\beta,j}}{b_j(n_{\alpha,j}+n_{\beta,j}+1) - 2e_j(n_{\beta,j}+1)}(a - c_{\beta,j})\,. \tag{14}$$

Then, given that equations are symmetric within each firm type, quantities at variant *j* are determined accordingly: if firm *i* is an *α-type*, its production level at time *t* would be $q_{i,j,t} = Q_{\alpha,j,t}/n_{\alpha,j,t}$, while if it is a *β-type*, its production quantity would be $q_{i,j,t} = Q_{\beta,j,t}/n_{\beta,j,t}$. Notice that if $n_{\alpha,j,t} = n_{\beta,j,t} = 1$, $b_j = 1$and $e_j = 0$, the production quantities are reduced to the expression (*a* - *c*)/3, which corresponds the optimal production amount in a two-firm, one-shot Cournot model with a homogeneous production cost, *c*. In order to bound quantities only to positive values, from equations (13) and (14) it can be noticed that a constrain on *e* is $e < 1/2$. Notice that an additional constrain is $a - c_{\beta,j} > 0$.

Thus, Cournot competition as just described above takes place at every product variant but firms can simultaneously serve several variants. Also, overcrowding of firms in a given variant might force firms to open up new variants if the one-time cost of bringing in a new product variant is lower than the expected benefit.

Firms also face scope diseconomies. One way to represent scope diseconomies is through niche width costs: the wider apart the firm's variant locations are, the higher the costs become. This implies that serving very heterogeneous audiences is costly. Let us also assume that *N* indicates the niche-width distance unit cost. Also, let us assume that $s_{H_{i,t}}$ represents the largest *Hamming (block) distance* between any two variants' location in firm's "niche" $H_{i,t}$ at time *t*. That is, the block distance corresponds to the largest compound distance-based dissimilarity per attribute between any pair of variants' locations in the set $H_{i,t}$. Of course, this is not the only possibility when it comes to measure distance between two variants. Other alternatives are also possible, like for instance the Euclidean distance or even computing the "largest shortest path" among variants. However, we believe it is more realistic having the firm computing the pairwise product variant dissimilarity per product attribute, and figuring out the compound largest distance of its served variants.

Therefore, $Ns_{H_{i,t}}$ represent the total niche-width cost. Therefore, firm *i*'s total profits and total production at time *t* are represented by

$$\pi_{i,t} = \sum_{j \in H_{i,t}} \pi_{i,j,t} - Ns_{H_{i,t}} , \tag{15}$$

and

$$q_{i,t} = \sum_{j \in H_{i,t}} q_{i,j,t} , \tag{16}$$

respectively.

It is noteworthy that the profit function (Equation 15) is fully separable. That is, the profit firm *i* gets equals the sum of profits obtained in every variant where the firm has presence (i.e., its niche). Nonetheless, it is important to highlight that competition is "networked" or nested. It means that the structure of competition in a given variant (in terms of total quantity, number of competing firms and price) is affected by a firm located in another variant that decides to either "invade" such a variant or open up a new one. Also, creation of new variants triggers a redistribution of demand across the existing variants in the space. Next, we will explain how firm expansion takes place and how the attribute space is shaped by such an expansion.

2.3 Firm expansion dynamics

Firms may decide either to enter competition in existing variants or to give birth to a new variant. Firms expand according to incremental profit expectations. At time *t* firm *i* decides whether (i) to keep their served variants, (ii) to open up a new variant (whose location is selected randomly from the neighboring positions), (iii) to expand into an existing variant (also selected from the neighboring existing variants), (iv) or to drop an existing one. The firm chooses what brings in more profits.

Alternatives (i), (ii), (iii) and (iv) are all considered simultaneously, so the firm chooses the one with the highest profit expectations. First, if the firm keeps its current niche, its profit expectation for the next time step would be:

$$\tilde{\pi}'_{i,t+1} = \sum_{j \in H_{i,t}} \tilde{\pi}_{i,j,t+1} - Ns_{H_{i,t}} \ . \tag{17}$$

The term $\tilde{\pi}_{i,j,t+1}$ is the expected profit in market *j*, which simply corresponds to the realized profit in the previous time step. Second, if the firm opens up a new product variant, the market demand would redistribute over all already opened up (i.e., active) variants. Expansion implies that demand is redistributed across the existing variants. There is a number of ways to do that, but here we simply assume that demand *M* is equally distributed along all active cells (different distribution alternatives are possible, but we do not expect them to significantly influence model outcomes. See Garcia-Diaz et al. (2008)). Thus, after the appearance of a new product variant, the firm would expect to gain:

$$\tilde{\pi}''_{i,t+1} = \sum_{j \in H_{i,t}} \tilde{\pi}_{i,j,t+1} - Ns_{H_{i,t\cup\{d\}}} + \tilde{\pi}_p - K \ . \tag{18}$$

Here, the term $\tilde{\pi}_{i,j,t+1}$ corresponds to the realized profit in the previous time step (for the sake of simplicity, we are assuming the firm cannot estimate the redistribution of demand before a new variant is opened up). Note that $s_{H_{i,t\cup\{d\}}}$ has to reflect the computation of the Hamming distance of the set $H_{i,t\cup\{d\}}$, which includes the new variant *d*. *K* corresponds to the one-time cost of *opening a new variant*. The term $\tilde{\pi}_p$ represents (expected) monopolistic profits at the newly targeted variant.

Quantity value depends whether the firm is *α-type* or *β-type*. Generally speaking, if the firm is *β-type*, the monopolistic profits at a given cell are computed as follows (since the location of the new variant is irrelevant in this part, please note we avoid using sub-indices related to product variants, but the same early naming conventions apply):

$$\pi_p = (a - bq^*)q^* - cq^*, \tag{19}$$

where $q^* = (a - c_\beta)/2b$. If the firm is *α-type*, its profit would be:

$$\pi_p = (a - q^*)q^* - (c_\beta - eq^*)q^* \,, \tag{20}$$

where the production quantity is given by $q^* = (a - c_\beta)/(2(b - e))$.

The third choice is to expand into an existing variant. In such a case, firm *i*'s profits are computed as:

$$\tilde{\pi}''_{i,t+1} = \sum_{j \in H_{i,t}} \tilde{\pi}_{i,j,t+1} - Ns_{H_{i,t\cup\{d\}}} + \tilde{\pi}_{i,d,t+1}. \tag{21}$$

Again, the values of $\tilde{\pi}_{i,j,t+1}$ corresponds to the profit perceived along the served variants in the previous time step. The term $\tilde{\pi}_{i,d,t+1}$represents the profit the firm would expect to obtain should it enter such variant, *d*. For such a purpose, the firm uses either equation (13) or (14), depending on firm type. In addition, the firm also considers the current values of the number of firms at the target variant.

The fourth choice is when the firm wants to evaluate dropping (i.e., abandoning) a variant, whose (expected) profits would be computed as:

$$\tilde{\pi}''''_{i,t+1} = \sum_{j \in H_{i,t}-\{d\}} \tilde{\pi}_{i,j,t+1} - Ns_{H_{i-\{d\},t}} \,, \tag{22}$$

where *d* stands for the dropped variant. Once more, the value of $\tilde{\pi}_{i,j,t+1}$ corresponds to the profit expectation at variant *j* the firm perceives in the next time step. In order to choose a candidate variant to drop, the firm randomly picks up a variant.

According to highest expected incremental profits, every time step, the firm decides whether to keep its current product portfolio (equation (17)), to open up a new cell (equation (18)), invade an existing variant (equation (21) or to drop one (equation (22)).

At *t* = 0, the product space stands for a market with only one product variant. The maximum active space comprises 11 x 11 variants (from variant 0 to variant 10). We run the simulation for 50 time steps. This normally provides enough time to arrive to the maximal two dimensional space fully saturated with possible product variants. Yet, as we will see next, partially occupied spaces, and consequently, non-integer dimensionality numbers, are also possible.

2.4 Price behavior

Regarding price dynamics, we set coefficient *a* equal to total demand, $a = M$. The slope $b_{j,t}$ of the price equation is set so that the market is cleared when the price is zero. At time $t = 0$ the full market *M* is place in one sole variant, so it is cleared when $b_{j,0} = 1$ (that implies that the largest possible demand in a given product variant is the whole market itself). Given that the market *M* is split equally among the existing variants, for subsequent values of the slope we set $b_{j,t}$ equal to the number of existing variants *for all variants*, so we make sure the market gets cleared if the price goes down to zero. It is important to say that, while considering expansion into a new variant, the firm takes the last observed slope as an input estimate of the next time price equation.

2.5 Product space dimensionality

Generally speaking, attribute space models represent markets or market segments with finite space segments, spun by axes of finite lengths, each axis hosting a finite number of "scale elements". The numbers of "scale elements" along axes determine the number of "cells" in the space segment. These "cells" are elementary units of the representation: it is normally assumed that neither producers nor buyers perceive differences between products in the same cell. Increasing integer dimensionality normally involves a 'thinning out' problem (Péli and Nooteboom, 1999). A two-dimensional product space with 10 scale values along dimensions has 100 cells (10x10); if a third product attribute of 10 scale variants develops, then cell number climbs to 1000 (10x10x10). Assuming that total demand remains the same, average demand density divides by 10 at each dimension shift. Even if the third developing product attribute is dichotomic at the outset (e.g., a product possesses or not a new feature), demand density halves.

The thinning out effect as calculated above is normally strongly overestimated, however, since demand distributes unevenly most of the time. Demand distributions used to have some abundant center(s), reflecting typical customer needs, surrounded by vast domains of thin peripheral demand (Carroll et al., 2002). It is a mathematical fact that pace of cell proliferation is much stronger at the peripheries than at the center: the majority of new cells appear near the walls of the space segment after an upward dimension change (Bruggeman and Péli, 2014). The result is that a large, often overwhelming, proportion of cells remain empty in higher dimensional attribute spaces, a fact well-known for example in social stratification studies (Blau, 1977; Kolosi, 1988).

The number of empty cells, and so space patchiness, do not only increase because of demand limitations, but also because of market actors' cognitive limitations. Since Hotelling's famous paper on selling outlet allocation along the line (1929), 'address type' attribute space models normally assume continuous dimensions for product positioning (Lancaster, 1966; Salop, 1975; Irmen and Thisse, 1998; Péli and Witteloostuijn, 2009). This also involves assuming that sellers and buyers are not only able to distinguish between infinitely many product attribute combinations, but also that all combinations stand for possible and recognizable product variants. Since product innovation and marketing of new product variants are tedious processes loaded with cognitive aspects like customer learning and producer engagement in the targeted market segments, it is a rare marginal case when sellers and buyers know all combinatorially possible product variants at the given market. Clearly, filters are needed to eliminate feature combinations that stand for not yet offered, or not yet even recognized, commodity variants. Fraction dimension is a suitable measure for the degree of attribute space spottiness. Its application allows correcting for the spurious proliferation of product variants as new dimensions are introduced.

We use Mandelbrot's similarity dimension concept (1983:37) to define "fraction dimension". Assume a finite *n* (integer) dimensional Euclidean space segment. We can assume without mutilating our main argument having uniformly *m* cells or variants (scale elements) per axis (see how to release this constraint in the Appendix.). The type of scales – ratio, interval, ordering or categorical – is immaterial from the point of view of the definition. Without restricting generality, we can assume unity distance between neighboring variants, thus rendering the space segment under investigation an *n*-dimensional hypercube composed of m^n variants.

The fraction dimension *DIM* of the space with *V* number of active variants (product variants) from the m^n total is defined as:

$$DIM = \frac{\ln V}{\ln m} \tag{23}$$

Equation (23) yields integer dimension n as special case for fully saturated spaces:

$$DIM = \frac{\ln(m^n)}{m} \; = n \tag{24}$$

Thus, fraction dimension yields integer dimension values for fully saturated subspaces. This important property explains why not using simply the percentage of active cells as patchiness measure.

As mentioned above, our model resembles an evolving networked Cournot competition. Each firm is simultaneously involved in Cournot competition across several product variants. If we equate "cells" of the fraction dimension measure with variants, we can argue that the available demand of each variant changes according to the fraction dimension of the space. Thus, as seen in Figure 2, we conceive the attribute space as a set of interdependent (i.e., networked) market variants where competition takes place, each variant located in a spot in a space, and each networked variant structure having a fraction dimension measure.[2]

A depiction of the evolving variant structure and its corresponding fraction dimension measure is illustrated in figure 3.

[2] An alternative approach to networked product spaces is proposed by Hidalgo et al. (2007).

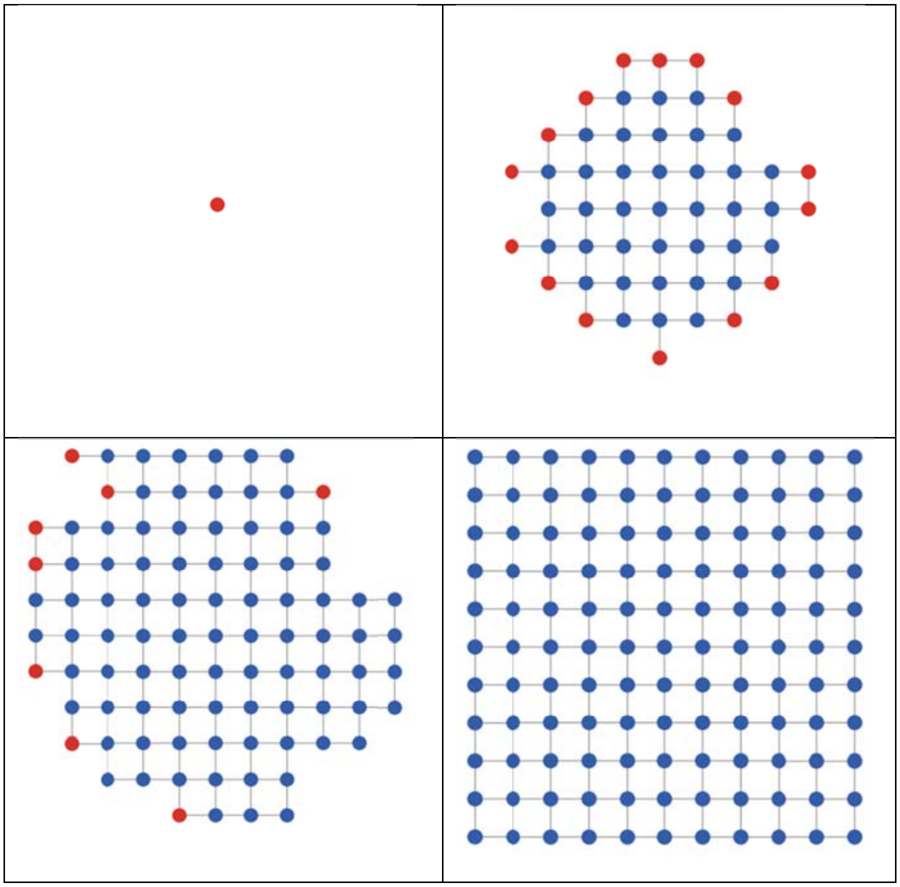

Fig. 3. Simulations that depict instances of four possible networked attributed spaces. Each cell (variant) represents a product variant, while links represent product variant interdependencies. In each variant, a Cournot competition takes place. From top left to bottom right, fraction dimensions are (i) 0, (ii) 1.65, (iii) 1.89, and (iv) 2, respectively. Blue variants represent established ones, while red variants correspond to newly created variants in the last iteration.

3 Summary of results

Next we report simulation results according to $N = 1000, K = 10000, M = 5000$ and $F = 100$. The simulation starts with one variant located at (5,5). Coefficient b_j and e are generated according to uniform distributions.

Figure 4 illustrates the evolution of the space dimensionality. At the beginning of the simulation there is only one variant, so the fraction dimension is zero. As new variants are created, the fraction dimension starts to rise. A fully saturated space is reached when the fraction dimension is 2.

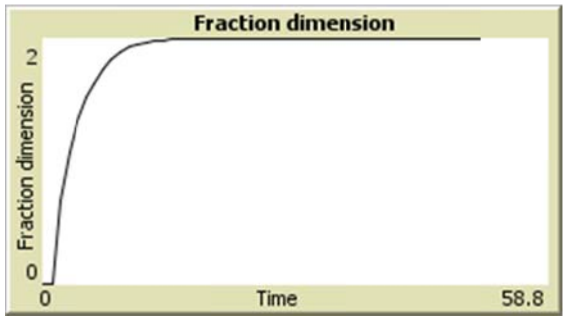

Fig. 4. Typical behavior of the space dimensionality over time.

Fraction dimension affects last iteration performance differently whether the firm is *α-type* or *β-type*. Figure 5 reveals that, as time (and dimensionality) increases, the last transaction profit is eroded for *α-type* firms while the *β-type* firms appear to reduce the gap with or even surpass the performance of *α-type* firms. This indicates that increasing dimensionality affects firms differently as to their scale advantage. Firms with limited scale advantage may be favored with product heterogeneity (something that has been observed in resource-partitioning settings. See Boone et al, 2002). Surprisingly, it appears that in many instances *α-type* firms are the ones that push the creation of new variants (as an attempt to exercise price discrimination). See Figure 6.

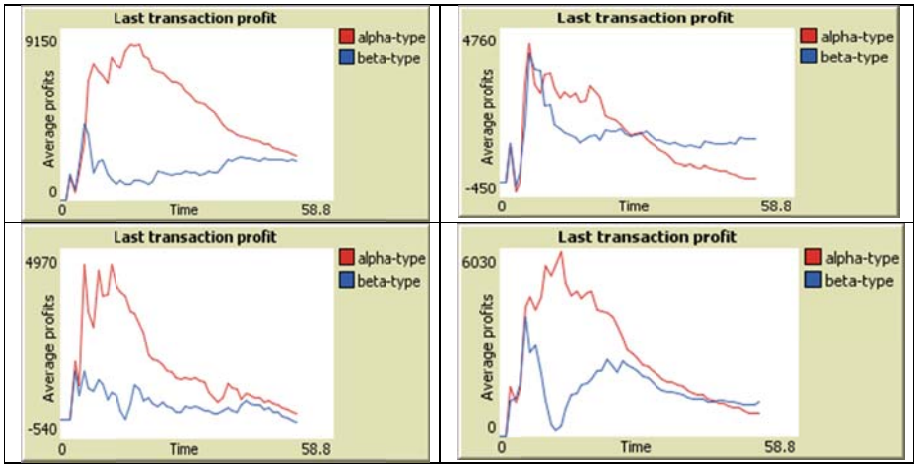

Fig. 5. Last transaction profit behavior over time

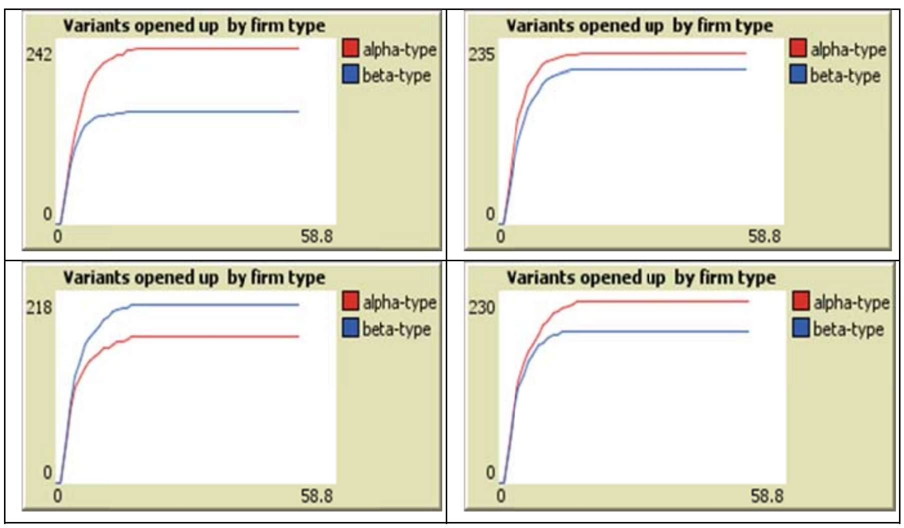

Fig. 6. Variants opened up by firm type (note: if two firms decide to create a variant in the same location, the number of new variants is counted as two).

At every dimensionality value, firms with scale advantage (*α-type* firms) register higher cumulative profits (and wider niches) than those with limited scale power (*β-type* firms), which reflects that scale dominance is sustained at all times. However, a straightforward conclusion is that increasing the number of attributes lessens the competitive power of scale dominance (Figure 5) and grant players with limited scale advantage a relative advantage. See Figures 7 and 8.

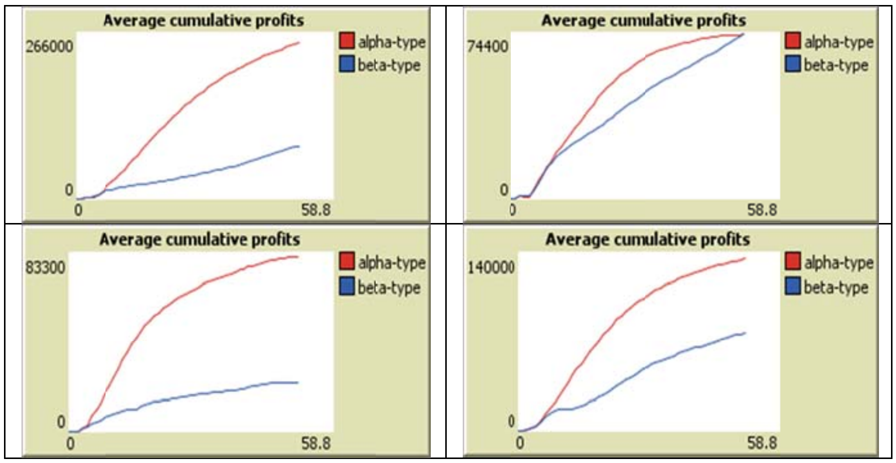

Figure 7. Average cumulative profits by firm type.

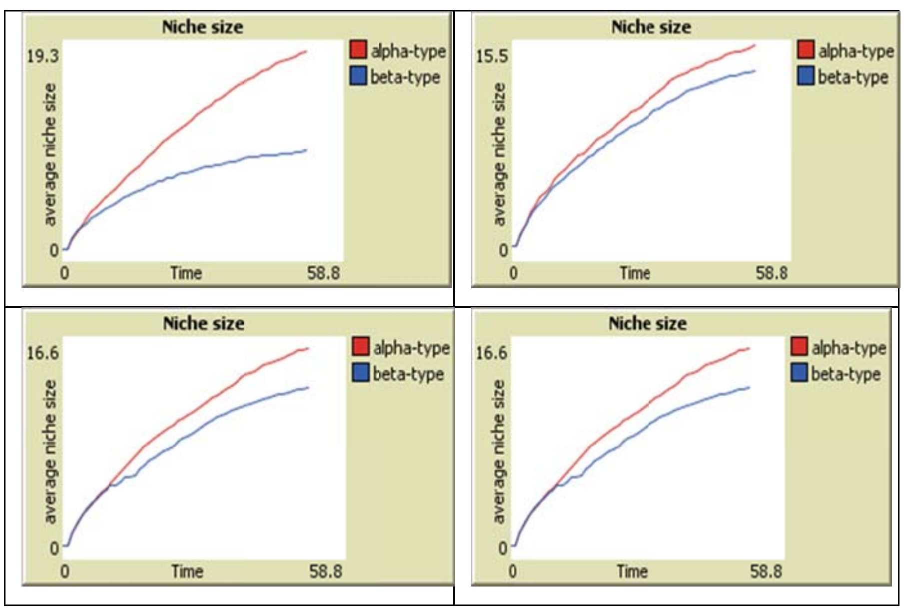

Figure 8. Average niche size (Hamming distance) by firm type.

Last, we observe the evolution of average costs over time. There is a point where costs appear to be lowest for both types, which implies there is an optimal fraction dimension figure for minimum costs. As the market unfolds, costs increase for all firms due to the weakening of the scale effect (the demand spreads over all variants) and the increasing niche-width costs. Even with increasing costs firms are able to charge higher prices, which bring about positive profits that gradually level off (see Figure 8).

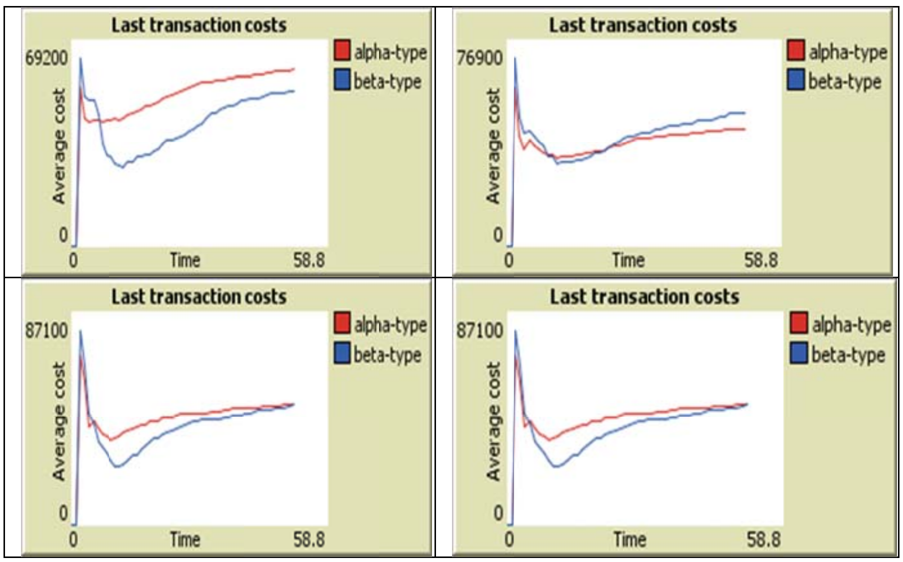

Figure 9. Average last transaction costs by firm type.

Conclusions

The model presented in this paper exemplifies one specific instance of co-evolution: that between the firm and the product space. Since the space gradually unfolds we find it convenient to represent its state according to a non-integer dimensionality measure that we label “fraction dimension”.

The model depicts the competition dynamics between two firm types: one type that is endowed with scale economies, one that is not. Increasing dimensionality affects firm types differently and in a nonlinear fashion. Firm with scale advantage seek price discrimination through the creation of new variants, at the expense of weakening their own scale advantage. Firms with limited scale advantage may find an increased instantaneous performance through increasing product variety. In line with ecological theories of markets, this set of results illustrates how firms with limited scale advantage might proliferate in a market with scale dominance through increased dimensionality.

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Supplementary material 1

Conceptual model conventions	Explanation	Range of values
M	Total number of consumer units	5000
F	Total number of firms	100
c_0	Minimum attainable unit cost	0
$c_{\alpha,j,t}$	α-type firms unit production cost	Variable
$c_{\beta,j}$	β-type firms unit production cost	$[a, e_j M]$
e_j	Unit cost reduction impact per unit of production quantity at variant *j*	Variable
$\pi_{i,j}$	Profit of firm *i* at variant *j*	Variable
P_j	Price of variant *j*	Variable
$q_{i,j,t}$	Firm *i*'s production quantity at variant *j* at time *t*	Variable
$q_{i,t}$	Total firm *i*'s production quantity time *t*	Variable
a	Price equation intercept	M
$b_{j,t}$	Price equation slope	Variable
n_j	Total number of firms at variant *j*	Variable
Q_j	Total quantity at variant *j*	Variable
$n_{\alpha,j,t}$	Number of α-type firms at variant *j* at time *t*	Variable
$n_{\beta,j,t}$	Number of β-type firms at variant *j* at time *t*	Variable
N	Fixed cost of attending a variant	[0, 1000]
$H_{i,t}$	Firm *i*'s niche at time *t*	Variable
$q_{i,t}$	Firm *i*'s total production quantity at time *t*	Variable
$\tilde{\pi}'_{i,t+1}$	Firm *i*'s profit expectation at time *t*+1	Variable
$\tilde{\pi}_{i,j,t+1}$	Firm *i*'s profit expectation at variant *j* at time *t*+1	Variable
$s_{H_{i,t}}$	Largest Hamming distance between any two variants in firm *i*'s niche at time *t*,	Variable

	$H_{i,t}$	
$\tilde{\pi}''_{i,t+1}$	Firm *i*'s profit expectation at time *t*+1 after inclusion of a new variant	Variable
V_t	Number of active variants at time *t*	Variable
$\tilde{\pi}_p$	Firm *i*'s profit expected monopolistic profits at new variant	Variable
$\tilde{\pi}'''_{i,t+1}$	Firm *i*'s profit expectation at time *t*+1 after expanding to an existing variant	Variable
$\tilde{\pi}''''_{i,t+1}$	Firm *i*'s profit expectation at time *t*+1 after a variant has been dropped	Variable
d	Dropped or additional variant	Variable
K	One-time cost of opening up a new variant	[0, 10000]

Table SP1. Variable and parameter definitions

Supplementary material 2

Fraction dimension with unequal number of scale elements (cells) per dimension

If the *n*-dimensional frame space has m_i scale elements along the i^{th} dimension, then the V_0 number of unit cells in the frame space is:

$$V_0 = \prod_{i=1}^{n} m_i \ . \qquad \text{(SP1)}$$

Therefore, it is suitable to apply at the generalized similarity dimension definition the geometric middle of the m_i values in place of the unique *m* value in Eq. (23):

$$DIM = \frac{\ln V}{\ln \sqrt[n]{\prod_{i=1}^{n} m_i}} \ , \qquad \text{(SP2)}$$

where the geometric average of the m_i values take the place of the unique *m*. Attribute spaces without an active element ($V = 0$) have no similarity dimension, while spaces with a single active element have zero similarity dimension according to Eq. (23).

Equation (23) can be transformed as:

$$DIM = \frac{\ln V}{\ln \sqrt[n]{\prod_{i=1}^{n} m_i}} = \frac{\ln V}{\frac{1}{n}\sum_{i=1}^{n} \ln m_i} = \frac{n \ln V}{\sum_{i=1}^{n} \ln m_i}. \qquad \text{(SP3)}$$

The generalized similarity dimension value is identical to Eq. (23) whenever $m_i = m$ for all i.