Modeling Competitive Beliefs on Social Network

William Leibzon*

October 15, 2015

Abstract

Activities in human society require coordination of beliefs to be successful. This paper explores two models of learning and coordination of beliefs on abstract scale-free social networks using agent-based network model. Beliefs are taken as an abstract set of either two (A,B) or three (A,B,N) elements and agents update their beliefs using either majority rule or coordination game dynamics, taking into account strength value of each belief. At the start of simulation only limited number of agents are seeded with beliefs and simulation explores how initial distribution effect the diffusion and final outcome. A notion of vulnerability and strength of a community of nodes is introduced to explain why stronger belief can not always win. Further by varying strength of beliefs and centrality of seeded nodes we can conclude that on a scale-free networks beliefs with smaller strength can propagate and win over the network if centrality of originally seeded nodes is higher, this corresponds to that something less believable or even not true can spread and win in the society if it is supported by more influential people. Additional results from coordination game based dynamic indicate importance of intermediate "Neutral" belief that continues to exist on the network even if one of the other beliefs does not survive, and which presence speeds up convergence and diffusion process.

1 Introduction

Social structure of human society is a result of interactions and communication. As it is impossible for any one person to know and communicate with everyone in the society, our interactions take place within a small circle made up of our relatives, friends, colleagues and others who we have a shared interests and activities with. Our interactions thus take place within human social network which can be represented as a graph where nodes are people and edges our connections to those we interact with. This network is how we learn and how we tell others what we have learned about the world. The coordination of

*Ph.D Student, Institute for Mathematical Behavioral Sciences, University of California, Irvine

human activities requires coordination within this network which can take form of cooperation or competition ([23], [14], [16])

Our knowledge can be represented as a set of beliefs about the world. This is what Buckland referred to as information-as-knowledge [2] and it can vary from simple and fairly unambiguous "my brother's name is Jack"[1] to complex topics of political nature such as "I believe that everyone should have equal access to health care." We acquire our knowledge and beliefs in form that Buchland referred to as information-as-thing [2] through direct observation of our environment, individual people we interact and communicate with, especially those we know well and communicate with on regular basis, and mass media and other forms of mass communication and information transmission like books. Direct observation is generally unambiguous, but it is not so for indirect information that is received through social network and mass media; and it is indirect information that accounts for majority of acquired knowledge.

If there is no or little prior knowledge and only one source of information that does not conflict with any prior knowledge, then new information serves to fill the gap and is acquired without much conflict [3]; in other cases the sources of indirect information maybe in conflict with each another or with prior held beliefs. For example one of your friends (or mass media source) may say that he or she believes government should be involved in providing health care to everyone, while your other friend may say that health care should be a strictly private business matter. If beliefs are in conflict, a person has to make a choice about who is right. These decisions are made based on how much one trusts each source of information, the strength of their statements, and how many people in one's network may have this belief [21].

How people make decisions as a group is something that requires considerably more research. Gigone and Hastie in their article ([9]) have noted that aggregate inference of a group directly relates to how many people are familiar with the question and are able to answer it. This is a majority-rule in group decision making, meaning choosing the answer that majority of the group believe in. Majority-rule has deep roots in human society and has been used since ancient times such as at Roman Senate, and continues to be used in majority of political bodies today. A capable mathematical model of group decision making is Group Social Decision Scheme (SDS) developed by Davis in 1973-1974 ([4]), it is a generalization of majority-rule that allows for more general schemes and can deal with cases where there may not be a majority but it is necessary to make a decision nonetheless. Latané thought that "impact should take the form of a power function, with the marginal effect of the Nth other person being less than that of the (N–2)th" [13]. But Sorkin and his co-authors found that just simple majority leads to better decisions than 2/3 or larger consensus ratio ([24]). In this paper we will use the basic majority-rule model since more complex models are not understood as well and since majority rule is most natural for abstract simulations where everyone is equal.

Having common beliefs also makes coordination easier, so people who want

[1] actually really not so simple as it requires knowledge of a concept of "brother" and naming

to coordinate try to convince others of their beliefs. Knowing who had been able to coordinate effectively may also be used as means to decide among conflicting beliefs, which is the basis of the second, game theoretic, model included in this paper.

Two type of networks have been recognized as important in approximating properties of real-life human social networks: Small World Networks and Scale Free Networks. In Small World networks average path between any two randomly chosen nodes is very small compared to large size of entire network, officially these are networks where average path has logarithmic relationship to size of the network [30]. Human networks have been shown to be small-world and this is where the concept of "six degree of separation" comes form; this concept means that for any two people anywhere in the world, there is a path of acquaintances that would connect them and it'd involve on average 5 other people.

Scale Free Networks are networks where degree distribution follows a power law - $P(k) = k^{\gamma}$where usually $2 < \gamma < 3$ (see [1]). In these networks there are very few extremely highly connected nodes and large number of nodes that have very few connections. Scale free networks are a subset of Small World networks since using highly connected nodes it is possible for information to traverse from one side to the other very quickly.

Real world network of human communication and social networks such as facebook are not quite scale free because almost everyone has more than just one or two connections and people with largest number of connections don't quite reach the huge number of friends expected in a pure scale-free network. Nevertheless several studies have shown scale free properties of real human networks and networks of human communication[15, 19, 20]. The degree distribution $P(k)$ in networks with scale-free properties is not described by clear power law but can be approximated by power law better than by a linear function. As we are interested in looking at behavior of human networks, we take scale-free networks as being the close abstract approximation for purpose of our models.

2 Models

Two types of models are used in the paper to explore competition and convergence of beliefs on an abstract social network. The models are explored computationally using simulations with software written as a custom plugin to Gephi[2].

Knowledge is is represented as two opposite conflicting beliefs A and B, and a special neutral belief N. People are represented as nodes on a network graph, and edges are their interconnections, such as friendship ties. Beliefs are public attribute of a node that is continuously shared with neighbors who may update their beliefs based on this attribute.

More formally a simulation model M = { V, E, B } and its state S are:

[2]Free software for doing statistical analysis and visualization of graphs. Available at http://www.gephi.org

- A set of Nodes$V = \mathbb{Z}_{\ltimes}$ ={ 1, 2, ..., n }

- A set of Edges $E \subseteq \{(v_1, v_2) : v_1, v_2 \in V\}$

- A set or Beliefs with strength values $B = \{(b, s) : b \in \{'A', 'B', 'N', 'U'\}, s = 0..\infty\}$

- An assignment of beliefs to nodes which is a dynamic state of the system: $S = \{(v, b) : v \in V, b \in B\}$

There are two different dynamics used for updating node beliefs corresponding to two models. The first model uses neighbor majority rule for belief updates while the second is based on coordination game.

Simulations are done using scale-free networks of 200 nodes generated using Barabasi-Albert model[1] and 200 node small world networks generated using Watts-Strogatz model[30]. Three nodes are originally seeded with one of the main beliefs A or B (3 nodes for A and 3 for B in a graph of 200 nodes) while others start with special U belief which has 0 strength. We are interested in which of the main beliefs are more likely to dominate over the network based on strength of the beliefs and centrality of original seeded nodes. Centrality here is in-degree/out-degree centrality which is based on number of edges node has, nodes with greater number of edges are considered to be more central as they can influence more of their neighbors.

The models being compared in this paper differ in the belief update rules and use or non-use of Neutral beliefs (i.e. if Neutral belief has strength 1 or 0). The strength of beliefs A and B are independent parameters varied across simulations.

2.1 Majority Rule based Model

The first model uses a majority rule dynamics. For each node the update rule is that during next turn a belief of the majority of node's neighbors becomes the node's belief. This is based on that people are more likely to believe something if majority of their friends believe in it. The weight of a beliefs A and B for node N at time t is calculated as:

$Neighborhood(n) = \{v_1, v_2, .., v_k\} = \{v : v \in V \; and \; \exists e \in E \; s.t. e = (v_n, v\}\}$

$NeighborBeliefs(n, b, t) = \{v : v \in Neighborhood(n) \; and \; (v, b) \in S(t)\}$ - where S(t) is state at time t

$WA_n(t) = size(NeighborsBeliefs(n, 'A', t))$ - number of nodes in the neighborhood of n with belief A

$WB_n(t) = size(NeighborsBeliefs(n, 'B', t))$ - number of nodes in the neighborhood of n with belief B

Minimum model involves only three beliefs - A, B and Unset. Beliefs are updated according to:

$B_n(t+1) = \begin{cases} 'A' : & WA_n(t) * SA > WB_n(t) * SB \\ 'B' : & WA_n(t * SA < TB_n(t) * SB \end{cases}$ - where SA is strength of 'A' and SB is strength of 'B'

In case of equality there is a random assignment of either 'A' or 'B'.

An extended model makes use of and assigns Neutral belief when weights are approximately equal:

$$B_n(t+1) = \begin{cases} 'A' : & 0.8 * WA_n(t) * SA > WB_n(t) * SB \\ 'B' : & WA_n(t) * SA < 0.8 * TB_n(t) * SB \\ 'N' : & otherwise \end{cases}$$

Only a few nodes are originally seeded with one of the main beliefs A or B (3 nodes each). We are interested in how "diffusion" may happen and which of the main beliefs would dominate or win over entire network based on which nodes were originally seeded and their location and centrality in the network, as well as based on strength of each belief.

This model is somewhat similar to standard information diffusion models [8]. But studies of networks and diffusion of information have generally looked at only one piece of information (or something else such as presence of infection) being distributed on the network with no competition involved. Diffusion in a case of competitive beliefs is the focus of this paper.

2.1.1 Similar Studies

There have been only few studies with competitive beliefs spread on the network. Here are the ones that are most similar:

- Rob Stocker and his co-authors have done a simulation study published in 2002 [25]where there were two opinions 0 and 1 randomly assigned to all nodes on the network with node-specific level of influence [3]and level of susceptibility values used to decide on how opinions are distributed on the network. Their model had updates of beliefs/opinions based on individual pairwise interactions and if level of influence of one node was greater than susceptibility of another, the opinion of dominant node was taken by both nodes. Such pairwise interaction model can be considered a game being played by nodes on the network, but the change immediately after interaction causes large number of updates and they reported great deal of instability in the results rather than consolidation of consensus for one of the opinions. This study looked at Random, Small World and Scale Free networks finding similarity between Small World and Scale Free.

- Tian Wang and co-authors have an upcoming paper "Analysis and Control of Beliefs in Social Networks" (currently available on archiv [29]) where beliefs are a real-valued node property ranging from -1 to 1 which are initially assigned at random and thereafter converge based on the beliefs of the neighbors. Controlled set of nodes contentiously broadcast their beliefs on the network causing change in other nodes. They have a set

[3]The simulation model developed by the author of this paper includes similar concept of trust. However trust concept is not node-specific but edge based. This is more realistic as different people often have entirely different opinion about if someone else should be trusted and how much. The trust was always set to 1 for what is presented in this paper. How this can be of use will be discussed in future research work.

of analytical results for Scale Free Networks (in their paper instead of directly using term scale-free they look at networks formed according to Preferential Attachment Model by Barabasi [1], as well as those formed according to Generalized Markov Graph model [7],[28]). Their results are very interesting as this is the only similar study with analytic results rather than simulations. However their results focus on control set and how to best choose these nodes using network graph adjacency matrix, concluding that best location are ones with high clustering coefficient of the nodes.

- Hashimzadea and co-authors have a paper in Journal of Economic Psychology on "Social networks and occupational choice: The endogenous formation of attitudes and beliefs about tax compliance" [10]. This is an experimental study that analyzed formation of groups with different beliefs on taxes which were based on professional association and modeled based on interaction of people on a social network.

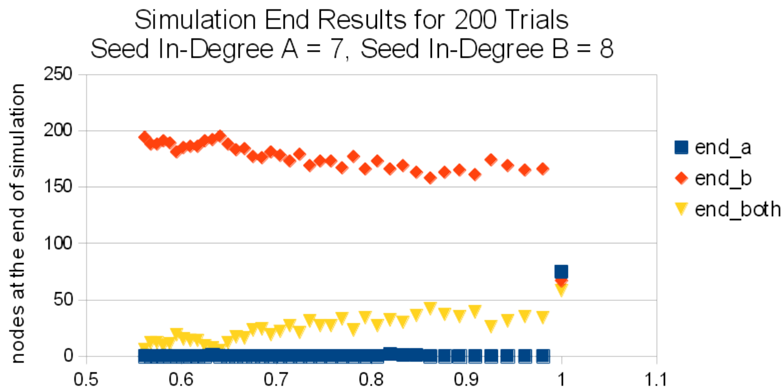
All these studies differ in various ways from what is described in this paper. But despite their differences, they all show importance of centrally located nodes for how competitive beliefs spread on the network.

2.1.2 Results from a Majority Rule model simulation and Discussion

The models are tested using computer simulations (software written in Java as a plugin go Gephi) and use several previously generated graphs of 200 nodes and about 800 edges (some graphs have 790 to 795 edges). Simulations are done as as a set of trials with same parameters - either 100 or 200 trials with same strength Sa, Sb for A and B beliefs and same probability for in-degree centrality for seeded nodes. Each trial starts with seeding of initial nodes (done using probabilistic algorithm that can seed beliefs A and B at nodes with expected difference in average centrality) and proceeds through turns during which nodes adjust their beliefs based on beliefs of their neighbors using majority rule. All nodes adjust their beliefs together during each turn. If all nodes in the network acquire same belief, the trial ends early, otherwise it continues for up to 200 turns as long as there are some updates still happening. The system records how many trials end with all nodes having belief A, or all nodes having belief B, or if both beliefs are present what is the ratio of A beliefs to B as average of all simulation trials.

Majority Rule Network Model - base results

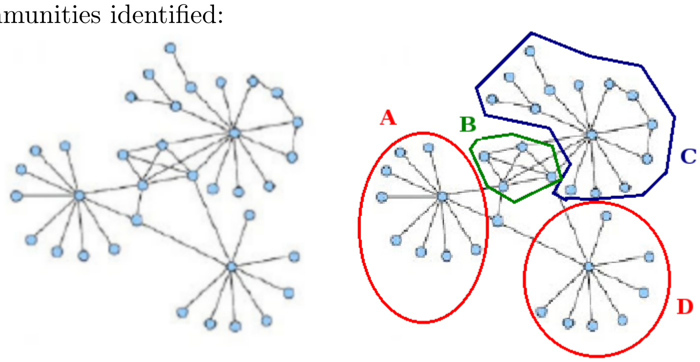
When strength of beliefs is varying between simulations, but centrality of initial seeded nodes is the same for both belief A and B, then result is:



The x axis above is SA/SB so when both SA=1 and SB=1 then approximately 33% of trials end with all nodes with A belief, 33% end with all B belief and 33% end with both. This means all these are equally probable given equivalent parameters for A and B beliefs. When strength of beliefs is varied even a little and SB>SA, and accordingly SA/SB < 1, then almost all trials end with all nodes with B belief.

Vulnerability and Strength of Communities

That some trials may still may have both beliefs even if one is stronger is a result of network structures. Especially important are communities, which are sets of nodes which all have more connections to other nodes in the community then to nodes outside of the community. These communities are complete or incomplete cliques (clique is a set of nodes where each node has connections to every other node in the clique). Here is an example of a graph with several communities identified:



Original Graph   4 Communities - A,B,C,D in the Graph

If one of the nodes in the community is initially seeded with certain belief while other nodes are Unset, the entire community quickly adapts this belief and can maintain it despite external pressure. Because even if all nodes outside have opposite belief with higher strength, the community nodes will stay with its belief because each node has more neighbors within community giving community believe higher weight according to majority rule algorithm.

More formally we can define vulnerability of the community to outside influence in terms of vulnerability of its nodes. This is based on how many connections the node has to outside as opposed to inside the community:

$Vulnerability(Node\, n|Community\, C) = \frac{size(\{e \in E: e=(n,v), v \notin C\})}{size(\{e \in E: e=(n,v), v \in C\})}$

$Vulnerability(Community\, C) = max(\{\forall n \in C: \; Vulnerability(n, C)\})$

In the above example:

$Vulnerability(A) = \frac{2}{8} = \frac{1}{4}$, $Vulnerability(B) = \frac{2}{3}$,

$Vulnerability(C) = \frac{3}{9} = \frac{1}{3}$, $Vulnerability(D) = \frac{2}{7}$

Numbers in this measure range from 0 to 1 and closer to 1 are communities that are more vulnerable to outside influence. The strength of the community can be defined as:

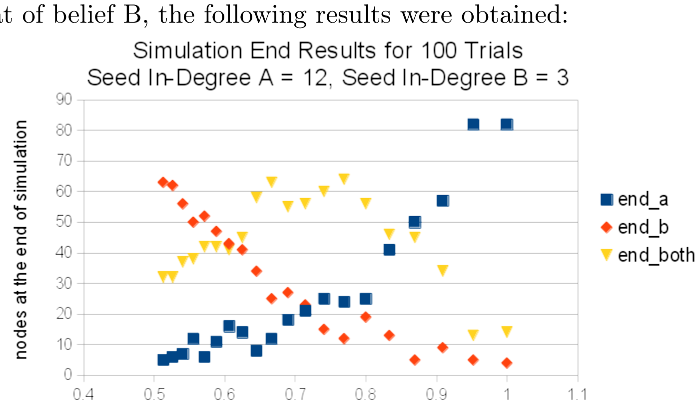
$Strength(C) = \frac{1}{Vulnerability(C)}$ - with values ranging from 1 to ∞

If all nodes in community C (from the figure above) had belief 'A', then strength SB of believe 'B' would need to be $SB > SA*Strength(C)$ or SA>3*SB in order for belief 'B' to penetrate and take over the community. Despite what above example may indicate, in the randomly generated graphs the communities with vulnerability < 0.5 are uncommon, which is why even small variations of strength can be enough for beliefs to propagate over the entire network.

Majority Rule Network Model - results with unequal centrality

There is a great deal of importance in regards to not only these communities, but how many nodes in a graph, get certain belief within first few turns. Even with higher strength of belief 'B', when initially seeded nodes for 'A' have more connections, then belief 'A' is able to spread to more of the Unset nodes making it more difficult for 'B' to win on the network.

When average centrality of nodes seeded with belief A was about 4 times that of belief B, the following results were obtained:



In above when strength of both 'A' and 'B' beliefs is the same, then higher centrality for 'A' gives it preference on the network and majority of trials end with all nodes having 'A' belief. With growing strength of 'B', the percent of trials in which A wins decrease until at abut SA/SB=0.75 an intermediate stage is reached where majority of trials end with both beliefs.

This confirms a generally well known fact that social networks and society in general are vulnerable to deception. If people who are important and very well connected in the society, say something, their opinion can spread over the entire society even if opposite opinion has more strength to it when directly compared against it. This is the base of propaganda and why it works and why media sources (which can be modeled as highly connected nodes with directional connections) are important for public opinion. Isolated communities and communities with strong internal beliefs, such as ones based religion, can be modeled as highly connected set of nodes with high community strength and low vulnerability; they are not as vulnerable to propaganda and to outside influence and can keep their internal beliefs together for a long time.

2.2 Coordination Game Based Model

2.2.1 Game Theory and Coordination Game

Game theory studies interactions of agents who can use multiple strategies and want to achieve best results. These agents, called players, engage each other and each gets a payoff that depends on a choice of its strategy and that of another player. Players are assumed to be rational and interested in maximizing their payoff.

Games that have been most studied are 2-2 simultaneous games with 2 players each with 2 strategies. These are represented in a 2x2 matrix - 2 rows for strategies of the row player, and 2 columns for strategies of the column player. Each cell in the matrix has two numbers - 1st is a payoff of a row player and 2nd of a column player. These payoffs maybe different for row and column players if they come from different sets, if not and payoffs are the same it is called a symmetric game. And because on a network row and column players are chosen from the same set of nodes on the network, in their 1-1 interactions they are playing symmetric games. Even with just 4 payoff numbers these games can serve as models for large number of phenomena and many interesting and important games have been studied: Prisoner's Dilemma, Hawk-Dove, Coordination Game, Stag Hunt (itself a type of Coordination Game) and others.

Model in this paper looks at basic Coordination Game. In this game players get high payoff if they both play the same strategy and low payoff if different. This can be represented by the following payoff matrix, where 'a' and 'b' are abstract payoffs for playing strategy A and strategy B:

	Strategy A	Strategy B
Strategy A	a,a	0,0
Strategy B	0,0	b,b

The equilibrium solution to the game is for players to play either (A,A) or (B,B). Even with a<b or b<a they are both Nash equilibrium solutions as deviation for either player leads to 0 payoff. The higher payoff equilibrium, lets say (A,A) is payoff-dominant while lower (B,B) is risk-dominant.

An extension of 2-person games are evolutionary games where large number of players (either finite or near infinite) play 2-2 games against each other. After each game, with some probability the player would re-evaluate results and may choose another strategy to play next time using specified update rule. The update rules fall into several common dynamics: best response dynamic, replicator dynamic, imitator dynamic, and others. Research topics are then what percent of players would have what strategy as times goes to infinity and how this relates to initial ratio of strategies. In basic evolutionary games any two players are equally likely to play against each other, if that is not the case the game is considered to have a network structure. For coordination game, it has been shown by Kandori that only risk-dominant solution is scholastically stable without additional network dynamics[11]. As Skyrms shows the presence of network dynamics allows to reach both equilibriums [23].

2.2.2 Games on Networks

A number of researchers have been interested in models where players are not paired to play randomly but instead there is a network structure and players can play only with some of the other players. These games can be roughly considered extension of evolutionary games, with update rules that make use of network structures. Some of these models allow for updates to network structure itself as part of the game. Here is a brief review of the research published in this area:

- Kirley looked at the Hawk-Dove game being played on sets of Random, Regular, Small-World and Scale Free networks. He provided results for propagation of hawks vs doves for all these networks showing that after large number of simulation turns Small World and Scale Free show similar results but games on small world networks take longer to stabilize. Overall his results are that Evolutionary Stable Strategies (ESS) do not always give best results for particular neighborhood and it depends on structure of a network[12].

- Shang and co-authors looked at evolutionary minority game on Random, Star, Regular and Scale-Free networks finding a number of network effects and evolution of global coordination [22].

- Ranjbar-Sahraei and co-authors looked at continuous action iterated prisoner's dilemma (CAIPD) where 2 players can choose strategies from continuous sets depending on how much they cooperate. They looked at games on Regular and Scale Free networks [18]

- Kevin Zollman in his dissertation on Network Epistemology[34] and related articles [33] looked at Cycle, Wheel and Complete networks and the effects of these networks on dynamics of social learning, which is modeled as probabilistic learning beliefs game. This model has some similarities to majority rule based model discussed earlier with competing beliefs that can spread through the network. He looks at networks with a very small number (10 or less) of nodes and shows that cycle structure results in quicker learning than wheel and complete graph networks.

- Payton Young looked at generalized coordination games (including stag hunt) on lattice networks, networks with small in-degree and out-degree, and several other network types[32]. He looked at these networks as a model for diffusion of innovation. He had a number of theoretical results, which in part show that if enough time is given for dynamic interaction a convergence is likely with result that depends on a network structure.
- Ebel and Borhnoldhave looked at Prisoner's Dilemma game being played on a network ([6],[5]). In their model network changes are allowed to find best neighbor. It shows emergence of a network with clusters and network structure that itself is similar to Nash equilibrium in a way that further adjustments would not lead to better payoffs. Their result is similar to results of Tomassini and Pestelacci discussed below.
- Tomassini and Pestelacci in several papers ([26],[17],[27]) look at coordination game being played on a network with rules that allow network updates based on edge-based "satisfaction" value (an average of payoff results over time). Lower satisfaction links are probabilistically removed and new connections can be established based on "introductions" from one's neighbors. The model starts with random network and over time evolves into a complex network with a series of clusters of nodes that play same strategy in a coordination game. At the end clusters merge together and what remains are two large clusters of nodes that all play the same strategy with only a few connections between the two clusters.

2.2.3 Model, Payoffs and Strategies

The model in this paper involves coordination game played on a scale free and small world networks. The motivation was to see if choosing those who can coordinate more effectively results in structures similar to majority rule model, and looking at if coordination can be used to decide among conflicting beliefs.

The base model involves three strategies - A, B and Unset. As with majority rule model 3 nodes start with A strategy, 3 with B strategy and rest are Unset. We are interested in how "diffusion" may happen and which of the main strategies would dominate in the network based on centrality of initial nodes with A or B strategies and how varying payoffs effects the game, which case can be compared to 2-player games where there is payoff-dominant equilibrium for following one strategy and risk dominant for following the other.

The simulation is divided into turns. During each turn, each node plays a coordination game using its current strategy against each of its neighbors. The payoffs from each game are summed up and then node's total payoff for the game is this sum divided by the number of neighbors: $P_n = \frac{\sum_{i \in neighborhood(n)} GamePayoff(S_n, S_i)}{size(neighborhood(n))}$

This is an agent-based model using imitative updates which for analytical results is considered equivalent to replicator dynamics in evolutionary games. At the end of a turn each node with 80% probability[4] may update its strategy

[4] 80% is probability leads to quick updates and makes it similar to every-turn update

by looking at payoffs of all its neighbors, and if any neighbor has a better payoff than the node would adapt strategy of that neighbor. If more than one neighbor has the same higher payoff, one of them is randomly selected to be imitated.

The base payoff matrix is:

	Unset	Strategy A	Strategy B
Unset	0,0	0,1	0,1
Strategy A	1,0	5,5	1,1
Strategy B	1,0	1,1	5,5

'Unset' strategy, that almost all nodes start with, is fully dominated by 'A' an 'B' which quickly spreads through the network and after several turns all nodes have either A or B. Further as result of the update mechanism more successful strategy could emerge as a overall winner. With above payoff matrix both strategies are equally likely to win, which is indeed the case for some simulations that have been run. What is of more interest is what happens when one of the strategies is weighted higher than the other. For this two parameters Str_A and Str_B are introduced making the payoff matrix:

	Unset	Strategy A	Strategy B
Unset	0,0	0,Str_A	0,Str_B
Strategy A	Str_A,0	5*Str_A,5*Str_A	Str_A,Str_B
Strategy B	Str_B,0	Str_B,Str_A	5*Str_B,5*Str_B

But as individual numbers only effect the scale, it is the ratio of Str_A/Str_B that matters and in the presented results ratio is what is plotted on the x axis.

The original model with only A, B and Unset strategies did not show cycles and other interesting effects that were seen with Majority Rule model. Therefore a second model was created that added new strategy N, as something intermediate between A and B. This strategy appears when node's best payoff neighbors have opposite strategies A and B and so its as if an agent can not decide for sure what to adapt and so chooses a middle neutral ground. The updated payoff matrix for the game with strategy N (but without Str_A and Str_B) is:

	Unset	Strategy A	Strategy B	Neutral
Unset	0,0	0,1	0,1	0,1
Strategy A	1,0	5,5	1,1	3,3
Strategy B	1,0	1,1	5,5	3,3
Neutral	1,0	3,3	3,3	3,3

Allowing for mixed strategies there are 6 equilibria in above game: (A,A), (B,B) which are the two payoff dominant strategy equilibria with payoff of 5 and (N,N), (N, 0.5 A + 0.5 B), (0.5 A + 0.5 B, N), (0.5 A + 0.5 B, 0.5 A + 0.5 B) which all have a payoff of 3. Here 0.5A + 0.5B means a player could choose a strategy of randomly, with 50% probability, choosing between A and B which gives the player average payoff of 3 equivalent to a payoff from playing pure N strategy. In evolutionary game not played on a network this predicts a possible

of majority rule model. Evolutionary Games models usually use lower probability to make strategy updates a smoothly changing curves which leads to very large number of turns for simulations to reveal equilibrium. Instead a number of simulations with same parameters are run here and results are averaged.

equilibria with some percent α of players who play N and $\frac{1-\alpha}{2}$ who play A and $\frac{1-\alpha}{2}$ who play B. But this is not a stable equilibria because if percent of players who play A is just slightly more than those who play B they will be imitated more and eventually everyone would be playing A.

Introducing again Str_A and Str_B gives the following complete payoff matrix:

	Unset	Strategy A	Strategy B	Neutral
Unset	0,0	0,Pref_A	0,Str_B	0,1
Strategy A	Str_A, 0	5*Str_A, 5*Str_A	Str_A, Str_B	3*Str_A, 3
Strategy B	Str_B, 0	Str_B, Str_A	5*Str_B, 5*Str_B	3*Str_B, 3
Neutral	1,0	3, 3*Str_A	3, 3*Str_B	3,3

This game eliminates mixed strategy equilbria that includes N if (Str_A+Str_B)>1 but as we shall see that is not the case for network games.

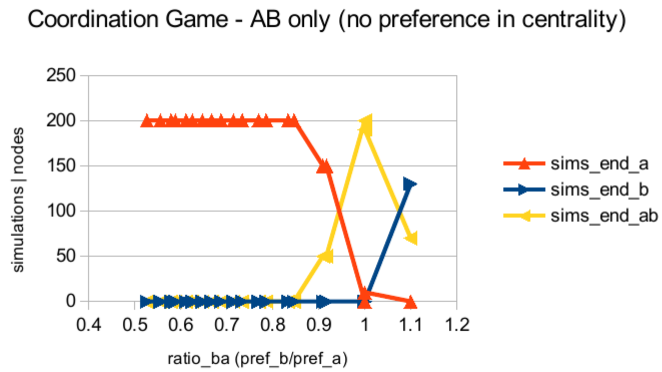
2.2.4 Results from a Coordination Game Model and Discussion

The simulations for this model were done same way as with majority rule model with graphs of 200 or 250 nodes. For this paper results from a specific graph with 200 nodes and 794 edges is used. As with the other model simulation consisted of 200 trials which had same Str_A and Str_B values and same probability initial strategy (seeded) nodes to have a centrality position. Each trial can run for up to 200 turns or may end earlier if every node is playing A or B strategies or if no nodes has changed their strategy in previous 2 turns. At the end it is recorded how many nodes are A, how many are B, how many N. Another trial then runs and at the end of a simulation it is recorded how many trials ended with all-A strategies, all-B strategies and the mix; how many nodes there were with A strategies, B strategies and Neutral is also recorded and averaged for all results. Independent variables that can be varied for simulations are Str_A and Str_B changing between 1 and 2 and centrality of initial nodes for strategies A and B. Simulations could either all include N strategy as discussed above or not include it.

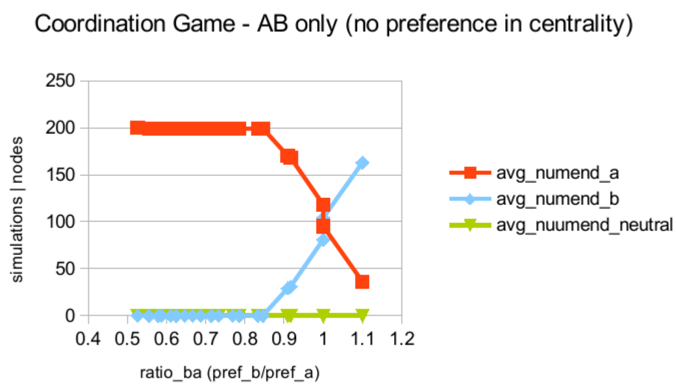
Results: Coordination Game Model without Neutral Strategy

In the first result there is no N strategy and centrality for strategies A and B set the same, so only Str_A and Str_B are varied. Plot below[5] shows how many trials, out of 200 for each set of parameters, ended with all nodes having A strategy (red graph), all nodes having B strategy (blue graph) and mix of both (yellow) :

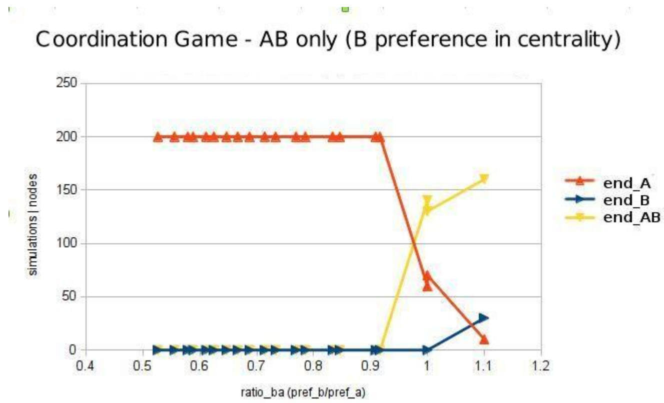
[5] On this and other plots Str_A is called Pref_A and Str_B is called Pref_B



Here the y axis is number of trials (on the graph these are called simulations) while x axis is a strength of strategy A over B, which is Str_A / Str_B. In simulations Str_A varied between 1 and 1.2 while Str_B grew from 1 to 2. As can be seen all simulations starting with Str_A / Str_B $<$ 0.8 end with all nodes with B strategy. With equal strength of A and B majority of trials end with both A and B, so for this case data on averaged percent of nodes that end with A and B strategies is relevant and shows that about equal number of A and B nodes at the end of a trial:

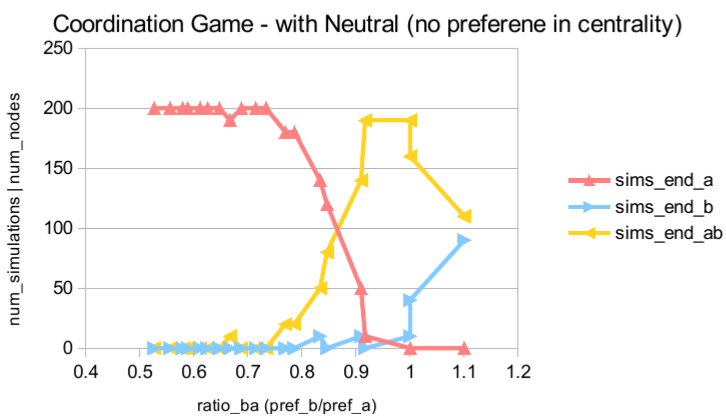


When centrality is varied such that initial strategy nodes for B have on average 6 times the in-degree/out-degree centrality of initial nodes with A strategy, then strategies are no longer equal and we see that strategy B with higher centrality is able to dominate the other strategy even when Str_A $>$ Str_B. This is the same result previously seen with the majority-rule model but with coordination game, these effects are much weaker:

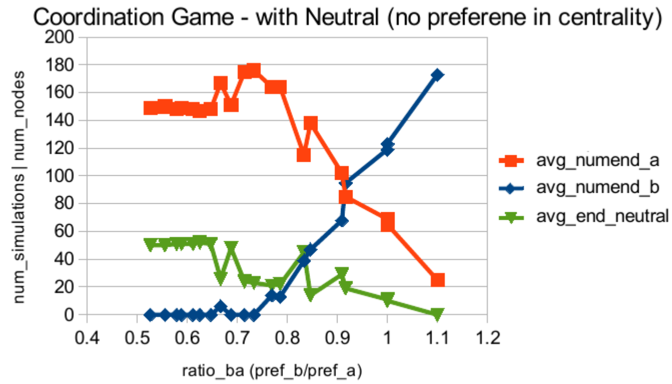


Results: Coordination Game Model with Neutral Strategy

We now go to results of model that includes Neutral strategy. As with no neutral, the first plot shows how many trials, out of 200 for each set of parameters (Str_A and Str_B), ended with all nodes having A strategy (red graph), all nodes having B strategy (blue graph) and mix of both (yellow):



While this appears to be similar to results with no neutral strategy simulations, with neutral for strategy A to entirely dominate it needs higher Str_A/Str_B ratio (~1.35=1/0.73, without neutral it was ~1.1). When looking at the average percent of nodes that ended simulation with A strategy (red), B strategy (blue) and Neutral (green) the results are even more interesting:



It appears that Neutral strategy survives and there are nodes that end with that strategy even when there is a large difference in strength of A and B strategies. This is similar to results that Peyton Young got [31] where in simulations he also had 3 strategies with intermediate surviving even when one or the other would not.

Another interesting result is that simulations with Neutral strategy actually converged faster rather than slower as might be expected. Cycles at the end also only occurred with Neutral strategy and in fact were quite common for equal strength of A and B strategy scenarios.

So Neutral strategy appears to bring more dynamics into the system. If there was more than just one intermediate strategy these dynamics could be even more noticeable, which maybe the case with real world social networks that have many variables.

3 Conclusion

The competition of beliefs is a more realistic model of distribution of information and knowledge than standard information diffusion models that only look at spread of one piece of information, so models described in this paper provide an insight about what is going on with distribution of beliefs and convergence to certain belief.

The models show importance of the network structure in the distribution of information and importance of centrally located people that can greatly influence beliefs of everyone else on the network. Result of the simulations show that beliefs that are stronger may not always win if those who have opposite view have a more central role in a social network. In fact by varying strength of beliefs and centrality of seeded nodes we can conclude that beliefs with smaller strength can win on the network if centrality of originally seeded nodes is much higher. This corresponds to that something less believable or even not true can spread and win in the society if it is being argue for by most influential and well connected people.

A notion of vulnerability and strength of a community of nodes is also introduced to explain why there can exist clusters that do not change to majority view in presence of strong belief. This is another common feature we see in the society with religious communities such as Amish. When these communities are closed and members maintain ties only within the communities, they are able to sustain their beliefs within larger society that has different norms.

By testing with model that uses coordination game and obtaining similar results to majority-rule based models we can conclude that information propagation is a sort of coordination. Centrality of seeded nodes appears to be less important, but same overall results still hold.

Additional results from coordination game indicate importance of intermediate "neutral" belief, which presence speeds up convergence and diffusion and which continues to exist on the network even if one of the other beliefs does not survive. This is unusual results which has been noted by others but is not well understood. But complexity of human society requires looking at many more than just 2 strategies and this topic definitely requires a lot more research. Simulations such as the ones described in this paper will be invaluable tool here because it is not likely that we can achieve analytic results in complex multi-strategy space.

This is ongoing research and what are presented here are very early results of simpler models. The simulation engine written is planned to be released as open-source tool and will allow people to create and run simulations on a variety of networks and specify different dynamics for game-theoretic models. The author is grateful for the support provided by the Institute for Mathematical Behavioral Science at UC Irvine to work on this research project.

References

[1] A. L. Barabasi and R. Albert. Emergence of scaling in random networks. *Science*, 286:509–512, 1999.

[2] Michael K Buckland. Information as thing. *JASIS*, 42(5):351–360, 1991.

[3] Michelene TH Chi. Three types of conceptual change: Belief revision, mental model transformation, and categorical shift. *International handbook of research on conceptual change*, pages 61–82, 2008.

[4] Davis James H. Group decision and social interaction: A theory of social decision schemes. *Psychological Review*, 80(2):97–125, 1973.

[5] Holger Ebel and Stefan Bornholdt. Coevolutionary games on networks. *Phys. Rev. E*, 66:056118, Nov 2002.

[6] Holger Ebel and Stefan Bornholdt. Evolutionary games and the emergence of complex networks, November 2002.

[7] Ove Frank and David Strauss. Markov Graphs. *Journal of the American Statistical Association*, 81(395):832–842, September 1986.

[8] Eric Gaussier. Models of Information Diffusion in Social Networks. In *Proceedings of the Second Symposium on Information and Communication Technology*, SoICT '11, pages 2–2, New York, NY, USA, 2011. ACM.

[9] Gigone Daniel and Hastie Reid. The impact of information on small group choice. *Journal of Personality and Social Psychology*, 72(1):132–140, 1997.

[10] Hashimzade Nigar, Myles Gareth D., Page Frank, and Rablen Matthew D. Social networks and occupational choice: The endogenous formation of attitudes and beliefs about tax compliance. *Journal of Economic Psychology*, 40(0):134–146, feb 2014.

[11] M Kandori, G.J Mailath, and R Rob. Learning, Mutation, and Long Run Equilibria in Games. *Econometrica*, 61(1):29–56, 1993.

[12] Michael Kirley. Complex networks and evolutionary games. *Complexity International*, 12, 2005.

[13] B. Lantané. The psychology of social impact. *American Psychologist*, 36:343–356, 1981.

[14] E Lieberman, C Hauert, and M A Nowak. Evolutionary dynamics on graphs. *Nature*, 433(7023):312–316, January 2005.

[15] Muchnik Lev, Pei Sen, Parra Lucas C., Reis Saulo D. S., Andrade Jr José S., Havlin Shlomo, and Makse Hernán A. Origins of power-law degree distribution in the heterogeneity of human activity in social networks. *Sci. Rep.*, 3, may 2013.

[16] Martin A. Nowak. Five Rules for the Evolution of Cooperation. *Science*, 314(5805):1560–1563, 2006.

[17] Enea Pestelacci, Marco Tomassini, and Leslie Luthi. Evolution of Cooperation and Coordination in a Dynamically Networked Society. *Biological Theory*, 3(2):139, 2008.

[18] Bijan Ranjbar-Sahraei, Haitham Bou Ammar, Daan Bloembergen, Karl Tuyls, and Gerhard Weiss. Evolution of Cooperation in Arbitrary Complex Networks. In *Proceedings of the 2014 International Conference on Autonomous Agents and Multi-agent Systems*, AAMAS '14, pages 677–684, Richland, SC, 2014. International Foundation for Autonomous Agents and Multiagent Systems.

[19] Markus Schläpfer, Luis M. A. Bettencourt, Sebastian Grauwin, Mathias Raschke, Rob Claxton, Zbigniew Smoreda, Geoffrey B. West, and Carlo Ratti. The scaling of human interactions with city size. *J. R. Soc. Interface 11 (98), 20130789 (2014)*, July 2014.

[20] Schneeberger Anne, Mercer Catherine H., Gregson Simon A. J., Ferguson Neil M., Nyamukapa Constance A., Anderson Roy M., Johnson Anne M., and Garnett Geoff P. Scale-Free Networks and Sexually Transmitted Diseases: A Description of Observed Patterns of Sexual Contacts in Britain and Zimbabwe. *Sexually Transmitted Diseases*, 31(6), 2004.

[21] Glenn Shafer. *A Mathematical Theory of Evidence*. Princeton University Press, Princeton, 1976.

[22] Lihui Shang and Xiao Fan Wang. Evolutionary minority game on complex networks. *Physica A: Statistical Mechanics and its Applications*, 377(2):616–624, April 2007.

[23] Brian Skyrms and Robin Pemantle. A dynamic model of social network formation. *Proceedings of the National Academy of Sciences*, 97(16):9340–9346, 2000.

[24] R.D. Sorkin, S. Luan, and J. Itzkowitz. Effect of Majority Rule and Initial Bias on Information Aggregation by Groups. *AFOSR/NSF/EADM Workshop on Information Aggregation in Groups and Individuals Decision Making*, 2003.

[25] Rob Stocker, David Cornforth, and Terry Bossomaier. Network Structures and Agreement in Social Network Simulations. *J. Artificial Societies and Social Simulation*, 5(4), 2002.

[26] M. Tomassini, E. Pestelacci, and L. Luthi. Social Dilemmas and Cooperation in Complex Networks. *Int. J. Mod. Phys. C*, 18:1173–1185, 2007.

[27] Marco Tomassini and Enea Pestelacci. Coordination Games on Dynamical Networks. *Games*, 1(3):242–261, 2010.

[28] Tian Wang, H. Krim, and Y. Viniotis. A Generalized Markov Graph Model: Application to Social Network Analysis. *Selected Topics in Signal Processing, IEEE Journal of*, 7(2):318–332, April 2013.

[29] Tian Wang, Hamid Krim, and Yannis Viniotis. Analysis and Control of Beliefs in Social Networks, January 2014.

[30] Watts Duncan J. and Strogatz Steven H. Collective dynamics of /'small-world/' networks. *Nature*, 393(6684):440–442, jun 1998. 10.1038/30918.

[31] H. Peyton Young. *Social Dynamics: Theory and Applications*. Handbooks in Economics Series. North-Holland/Elsevier, Amsterdam, 2006.

[32] H. Peyton Young. The dynamics of social innovation. *Proceedings of the National Academy of Sciences*, 108(Supplement 4):21285–21291, 2011.

[33] Kevin J. S. Zollman. Social structure and the effects of conformity. *Synthese*, 172(3):317–340, 2010.

[34] Kevin JS Zollman. *Network Epistemology*. PhD thesis, University of California, Irvine, 2007.