The spatial allocation of renewable power infrastructure

An economic assessment of energy landscapes with an agent-based modelling approach

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Abstract. The question how a least-cost spatial allocation of sustainable electricity infrastructure may look like using different decision-making procedures (markets, different kinds of land-use and grid regulations) has not yet been analyzed explicitly. We measure the sustainability of emerging energy landscapes providing power from renewable energy sources (RES) by an overall welfare function also comprising all kinds of space-related disutility, i.e. spatial externalities - be they site-specific or related to the distance to a consumer center. The presented agent-based model (ABM) concept aims at assessing different policy scenarios to govern the land-use for energetic purposes under the constraint of ensuring the electricity supply for a virtual landscape with RES. To derive "optimal" spatial allocation an agent-based modeling approach which includes a virtual landscape, three demand centers and profit-oriented producers of renewable power is implemented. For the design of the electricity grid and the calculation of grid-related reinforcement costs a load-flow model is applied being able to map also grid externalities during the RES expansion in space.

The model allows RES producers to choose profit-maximizing cells for plant installation until the given demand for power of the virtual landscape is met. Different policy scenarios allocate particular costs to agents (e. g. grid reinforcement costs, spatial externalities) or restrict the land-use with respect to ecological or social restraints. The overall efficiency of allocation (total cost level) as well as the distributional fairness (regional net costs) can be evaluated for the policy scenarios. In a first application we analyzed the effect of land-use restrictions polices regarding the installation of wind power plants.

Keywords: Agent-based modelling, Renewable energy sources, Externalities

1 Introduction

The German Federal governments' energy concept, adopted in 2010 (BMU/BMWi 2010), aims to cover 80% of the gross power requirement in Germany by renewable energies till the year 2050. Therefore the hitherto mainly centralized energy system that consists primarily of conventional power plants should be transformed in a more decentralized system that would be dominated by renewable energy sources (RES). As a consequence, new forms of energy landscapes (Blaschke et al. 2013) emerge and particularly the grid infrastructure has to be adjusted to the new "spatial needs" of RES, under the restriction of a limited available space and various competing spatial interests stemming from economic, social and ecological land-use.

Under the current policy setting in Germany with feed-in tariffs for RES the producers' decision where to locate RES infrastructures is dominated by natural locational factors unless there are land-use restrictions for plant installation. Accordingly wind conditions and solar radiations are the most important criteria in a multi-criteria site assessment of possible investors. Simultaneously it is important to harmonize the newly installed RES capacity with the existing grid infrastructure. Hence the increasing spatial gap between the demand (in the south and west of Germany) and the supply of electricity (in the North and East of Germany) can lead to higher grid reinforcement costs because of the growing risk of reaching critical grid capacities (Nolden et al. 2013).

Likewise social and ecological externalities of RES infrastructures as part of newly emerging "energy landscapes" have to be considered. They might be regarded as less important than externalities from the conventional electricity production (climate change, radiation), but the decentralized regional approach of RES power supply evokes new spatial disutilities (Meyerhoff, Ohl and Hartje 2010). Therefore the well-known "not in my backyard" (NIMBY) problem (Frey and Oberholzer-Gee 1997, Groothuis and Whitehead 2008, Wolsink 2007, van der Horst 2007) and negative environmental impacts of RES, e. g. bird mortality (Drewitt and Langston 2006, Eichhorn and Drechsler 2010, Kikuchi 2008) have to be taken into account. On the other hand regions may benefit from RES installations through local added values via taxes, local production, and local maintenance of projects involving RES (Hirschl et al. 2010).

Our aim is to derive “optimal” spatial allocations for electricity infrastructure under the constraint of ensuring the electricity supply for a virtual landscape with RES. Consequently the focus lies on the assessment of various policy scenarios with different market and land-use regulations to govern the spatial allocation for energetic purposes. Therefore a conceptual agent-based model (ABM) is implemented. These kinds of models attract growing attention in the scientific community as a useful modelling tool for spatial related issues, autonomous decision behaviour, and policy impacts. Yet, there already exist several ABM models with an energy-related background on a global (Chappin and Dijkema 2010) or local scale (Wittmann 2007) dealing, for example, with electricity markets (Bunn and Oliveira 2001; Krewitt 2011) or the general transition procedure within electricity systems (Ma and Nakamori 2009). However, these models either exclude spatial related issues, or focus on a particular case study without considering cross-cutting effects among regions. In this paper, the presented regional ABM-approach combines rational choice behaviour based on economic theory with land-use modelling originating in geography, and load flow modelling coming from electrical engineering. The approach allows the illustration of socio-economic (e.g. spatial desutilities, regional value creation) and techno-economic (e.g. grid infrastructure) aspects together in one model framework to estimate the efficiency and fairness of a sustainable energy system on a regional scale. We determine first results by testing the impacts of various land-use regulations on the model outcomes. For simplicity, we focus on a single RES technology only (say onshore wind power) with a given capacity per plant unit though neglecting any intermittency problems. The model was programmed in NetLogo (Wilensky 1999) and R (R DEVELOPMENT CORE TEAM 2013) whereby only the site assessment takes place in NetLogo. The two programs are connected via a link package developed by Thiele (2012).

2 The model

2.1 Entities, variables and scales

The objective of the agent-based model is the least-cost allocation of RES power infrastructure (power plants, grids) within a virtual landscape in order to guarantee a secure supply (in sense of market clearance aspects but without taken into account volatile production patterns) of given consumers' overall electricity demand in three settlements (as regional consumer centres) under consideration of distributional aspects of regional net cost burden. The central target function of the model is the overall welfare function W of the virtual energy landscape, comprising producer and consumer surplus from energy provision (PS, CS) and space-related external disutility X emerging from land-use for RES power production and transportation.

$$W = PS + CS - X$$

The cumulative profits π of the N RES producers represent the producer surplus PS. The producers have to consider their business production costs C_P, which represent all arising costs during the life time of a plant, as well as the business connection grid costs C_{GC}, to connect the installed plant with the existing grid infrastructure. Business production costs include all fix and variable costs like the investment, maintenance and repair costs which are all dependent on the plant type.

$$PS = \sum_{n=1}^{N} \pi_n$$

The consumer surplus CS consists of three parts. First we have the positive utility due to the purchase of electricity, calculated as difference between the consumers' willingness to pay $p_{max}$and the actual price for electricity p. Second, a multiplier effect m due to the regional electricity production out of RES is another part of the consumer surplus. It is defined as regional economic welfare effect because of RES installation, which for example emerges through tax revenues and maintenance companies. Third, the business costs for the existing grid system C_{GE} have to be subtracted from the aforementioned terms of CS. These costs emerge if net segments of the existing grid system have to be reinforced due to load flow changes because of newly installed RES plants. As the electricity demand of all consumer centres Y^D will be defined at the beginning of a model, any welfare improvement is performed via a total cost minimization.

$$CS = Y^D \frac{p_{max} - p}{2} + mY^S - C_{GE}$$

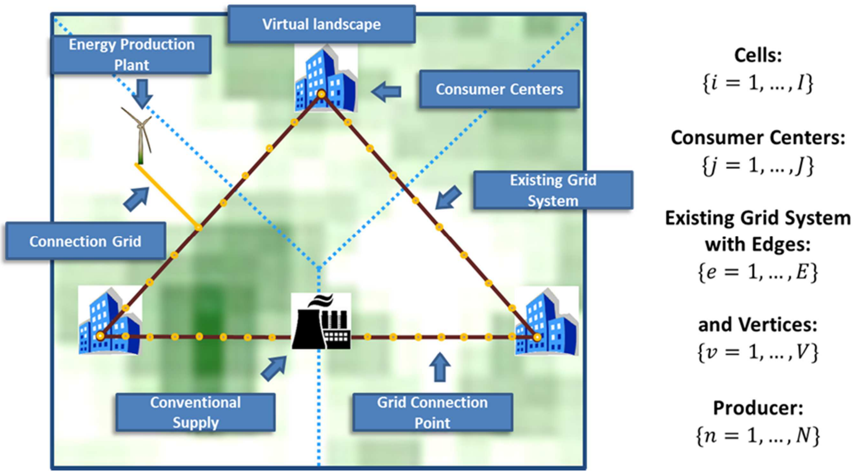
The third term X of the welfare equation is defined as the sum of external spatial costs (spatial disutility). These costs can arise due to NIMBY-related or ecological issues. Although we know that a precise spatial definition of both issues is hard to obtain, it is possible to generally divide them into a distance-related (van der Horst 2007, Eichhorn and Drechsler 2010) and a site-related cost component (Wolsink 2007). In the model both are captured cumulatively via a distance- and a site-dependent external cost function. After summarizing these two functions we get the total external spatial costs, which arise because of production infrastructure $X_P$and distribution infrastructure X_G (i. e. grids).

$$X = X_P + X_G$$

The site assessment of the RES producer takes place in a virtual landscape defined as a raster grid which is part of a higher market system. This means that the virtual landscape can be seen as a regional representation within national boundaries. Due to the high demand of RES plants for space, only one RES producer can be placed per cell. Natural location factors like wind conditions (soil quality, solar radiation etc.) are included in a simplified manner as energetic yield potential h. The sensibility of a site regarding the installation of a production or distribution infrastructure is defined as site-dependent external costs x_s. Ecologically important sites or landscapes with considerable aesthetic value suggest a high vulnerability regarding new constructions in general. The energetic yield potential and the site-dependent external costs are randomly distributed under consideration of the neighbourhood values of the cell, which can be explained due to the spatial correlation of wind conditions and land use types on a regional scale.

Figure 1 shows all relevant model entities. The three consumer centres, as illustrated above, are defined as conglomerations of households with a fixed (and equal) electricity demand. Simultaneously they act as a local administration entity with a particular manageable catchment area. They are affected by the external spatial costs that arise within their catchment area. On the contrary they benefit from the multiplier effect of the RES which also depends on the installed capacity within the catchment area of a consumer centre. To guarantee the supply security, all consumer centres are connected with each other via an existing grid infrastructure. The existing infrastructure is adapted to a conventional electricity supply from a single load point, defined as conventional supply.

Fig. 1. Abstract representation of all relevant model elements



New RES power producers have to connect their plants to the existing grid system via a grid connection point and a connection grid. If the existing grid system has to be reinforced, additional costs for grid expansions arise, which have to be paid by the consumer centres in the market scenario.

The actual location choice concerning new plants lies in the hand of the RES producers. They are profit oriented. In other words, producers act as rational agents following the merit order principle which means that the sites with the highest profits regarding the producer profit function will be picked first. In the market scenario no land-use restriction or internalization policy applies.

2.2 The model procedure

The objective of the consumer centre is to completely supply its defined energy demands out of RES, which means that a given conventional supply meeting the current demand will be incrementally substituted by a RES power supply. Accordingly, a new producer of renewables picks a site in the virtual landscape regarding his profit function during every time step. This procedure is repeated till the total RES supply within the virtual landscape is equal or higher than the total demand.

$$Y^S \geq Y^D$$

As mentioned earlier, the profit-oriented site assessment does not consider a catchment boundary of the consumer centres. Producers simply maximize their profit function

$$\max_{y^s, c_n} \pi(y^s, c_n) = p y^s - c_n \, .$$

The profit function depends on the particular policy scenario and its intended cost categories for the producers. In the basic market scenario the producer takes the business production costs and the business connection grid costs into account. They are represented in the grey box of figure 1.

$$c_n(c_P, c_{GC}) = c_P + c_{GC}(d_{iv}) \, .$$

Since all producers use just one single plant per definition, the business production costs c_P are equal to the factor costs of the respective plant type w_{PC}

$$c_P = w_{PC} \, .$$

For the calculation of the connection grid cost the minimal distance between the plant and the next connection grid point on the existing grid system will be calculated. Afterwards, it will be multiplied by a distance related factor price

$$c_{GC}(d_{iv}) = w_{GC} \min d_{iv} \, .$$

The energetic output per plant is the result of the multiplication of the energy yield potential per cell by the performance parameter of the plant l_{PC}. Therefore, y^s represents the average amount of electric work, which is transmitted to the consumer centres via the grid system. In the basic policy scenario the performance parameter is constant due to the usage of only one wind power plant type

$$y^s(h) = hl_{PC} \, .$$

Afterwards the energetic output per plant will be remunerated, dependent on the given average price for renewable electricity. No explicit electricity power market is implemented in the model. Instead, a fixed feed-in tariff scheme for the entire landscape applies.

Due to a lack of space all relevant sub models will be explained in detail within the addendum.

3 Policy experiments: The case of land-use restrictions

To test the impact of different policy scenarios which are defined as a set of rules of allocating welfare-relevant costs across actor in a particular we have designed policy experiments to test in particular the development of the welfare outcomes of the model. The use of ABMs to test the influence of policy scenarios in the context of land use change and agriculture activities has already been applied by Berger et .al (2006) and Happe et al. (2006). Several policy scenarios, that are associated with RES allocations, can be examined within the presented ABM framework. In general we can distinguish between a market-based and a regulation-based allocation framework.

3.1 Land-use restriction policies

In our first policy experiment we distinguish between a site- and distance-dependent land-use restriction with the concerning land-use restriction parameters lur_{x_d} and lur_{x_s} representing the restricted area share in % by taken into account the total available area of the virtual landscape. Depending on the defined value of lur_{x_d} all cells within a certain distance to the consumer centre will be restricted, whereas in the case of the parameter lur_{x_s} the cells with the highest site-dependent external costs will be excluded for RES production plants. The aim of the policy experiment is to measure the influence of increasing land-use restrictions on the total welfare after a model run is completed.

Accordingly, the land-use restriction parameters lur_{x_s} and lur_{x_d} will be systematically increased after every complete model run, till it is not possible anymore to supply the complete demand of the consumer centres with RES with the remaining cells. The installation of connections grids is only restricted at especially vulnerable sites (above the mean value of x_S), as installations of connection grids from the RES plant to the existing grid structure have to be guaranteed.

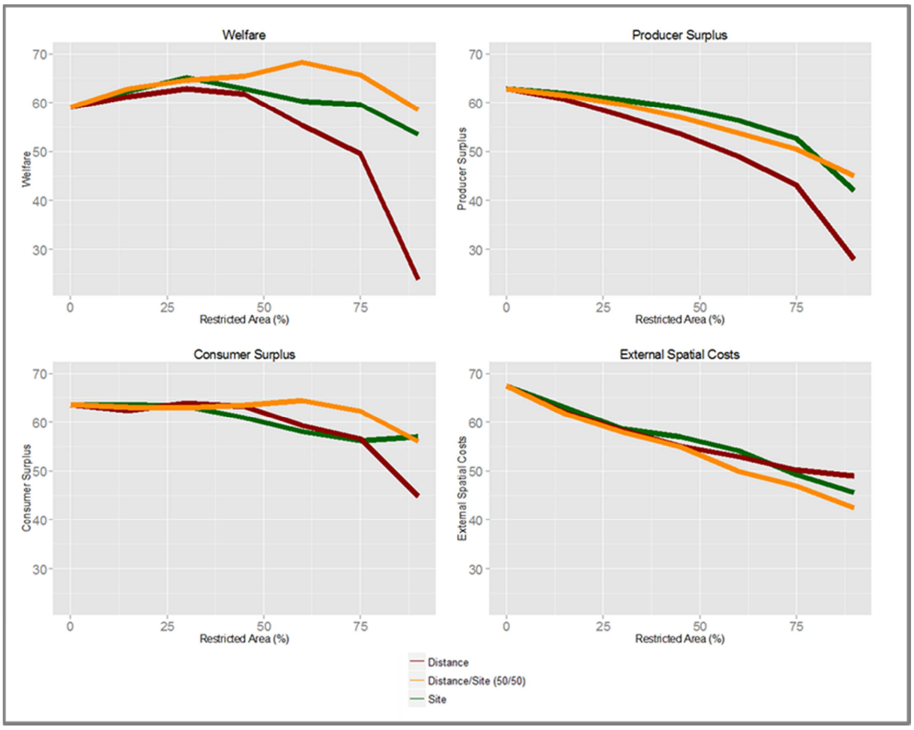
This process can be done either for a single land-use restriction type (site- or distance dependent) or for a combination of both. We distinguish between three policy scenarios in the following policy experiment: first the site-dependent land-use restrictions, second the distance-dependent land use restrictions and finally a combination of both, whereas the site-and distance-dependent land-use restrictions have an equal weighting. But before a reasonable parameter set for the model runs have to be defined.

3.2 First results

We discuss the results for a homogeneous demand structure, which means that all consumer centres have the same electricity demand. Figure 6 provides an overview on the development of the welfare per policy scenario by increasing land use restrictions. As mentioned before we distinguish between site-dependent land-use restrictions (dark green line), distance-dependent land use restrictions (dark red line), and a combination of both (dark orange line) with equal weightings.

A positive welfare effect for all policy scenarios can be observed in the first segment of the plot. These benefits are achieved due to a better internalization of external spatial costs. At the same time the average site productivity per wind power plant decreases which leads to a reduction of the producer surplus. This effect gets even stronger with growing area restrictions for all scenarios.

Fig. 2. The influence of land use restrictions polices on the welfare equation variables



Consequently the welfare decreases again after having reached its maximum below the level of the pure market scenario with no land use restrictions. Even the effect on the consumer surplus, while having high area restrictions rates, can be negative because of an increase of the reinforcement costs for the existing grid infrastructure. This is the consequence of decreasing concentrated production areas, due the properties of the landscape and the land-use restriction mechanism which vice versa lead to a centralized wind power production structure with all related demands for the existing grid infrastructure.

4 Summary and Conclusion

The presented agent-based model allows us to analyse the consequences on the spatial allocation of sustainable electricity infrastructures by testing a variety of policy scenarios. Therefore the model focuses on: (I) agent behaviour of producers and consumers, (II) overall costs function that represents all types of costs (including especially spatial externalities), and (III) the constraint of ensuring a given electricity supply for a virtual landscape with renewable energy sources. The performance of each policy scenario can be assessed under the aspect of overall efficiency of the virtual energy landscape and distributional fairness among regions. This model is based on an interdisciplinary discussion process linking economic theory to land-use decision modelling and load flow modelling in order to enhance the system understanding for allocation mechanism of electricity infrastructure.

First results have been obtained in the field of land-use restriction policies by examining a homogenous demand structure among the consumer centres. The effectiveness of site- and distance related restrictions have been tested with the result that neither a pure market solution without any land-use restrictions nor a simple conservation policy that only considers social and ecological costs can lead to an "optimal" solution for the society under the given assumptions. Furthermore a combination of the site- and distance-dependent land use restriction types is more efficient than any single applications. No significant differences regarding the fairness parameter have been observed. Therefore further experiments with changing consumer centre demands have to be executed.

In the future we would like to examine further market and regulation based policy scenarios. In addition to the presented regulation based experiments, the impact of explicit construction zones for RES designated by each consumer centre will be analysed. Subsequently the introduction of a market based reference yield model ("Referenzertragsmodell") and a producer's participation in the reinforcement costs will be tested.

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The Addendum

Sub-models

The Spatial Module.

All spatial related calculations, like the computation of the external spatial costs and the minimal distance for the connection grid, take place in the Spatial Module. Spatial externalities affecting directly human beings (e.g. landscape aesthetics, health effects: "social external costs") and spatial externalities affecting ecosystems with indirect impacts on human well-being (e.g. bird mortality, dissection of landscapes: "ecological external costs") are differentiated. They may be dependent either on the distance to the consumer centres or to the site-specific ecological vulnerability:

$$x_P = x_D + x_S$$

The sum of the calculated distance-dependent external spatial costs $\mathrm{x_D}$, and the randomly distributed site-dependent external costs $\mathrm{x_S}$ is the input $\mathrm{x_P}$ for the calculation of the external spatial costs for production and transportation. As land use types are often arranged in clusters, we determine a similar arrangement of the site specific ecological vulnerability by including neighbourhood values in the distribution process of $\mathrm{x_S}$

$$x_D = \sum_{j=1}^{J} \alpha d_{ij}^{-\beta} \frac{y_j^D}{\bar{y}_j}$$

The distance-related costs will be determined by the distance between a cell i and a consumer center j, an exogenous intensity parameter α and, a curvature parameter β within the interval $[0,1]$, which leads to an exponential decrease of the cost parameter with increasing distance We presume that the distance-related costs are constant across all consumer centres. Accordingly these costs also depend on the ratio between the electricity demand of a consumer centre $y_j^D$and the average electricity demand $\bar{y}_j$. Due to the cross-border visibility of RES plants, distance-related costs do not only arise in the consumer centre's catchment area where the plant is situated, but have to be considered in general for all affected consumer centres.

If a producer installs a RES plant on a particular cell, external spatial costs for both production and transportation of RES power arise. For the calculation of the external spatial costs for transportation, all cells which are crossed by the grid infrastructure have to be included. For that reason the least-cost path method (cf.Pinto and Keitt 2009) will be applied to find a least-cost route with regard to the external spatial costs between the cell of the RES production plant and the closest connection grid point.

In the end all resulting external costs will be aggregated. The parameter θ is included to determine the intensity differences between external spatial costs due to production and the transportation infrastructure. The relations between the aggregated external spatial costs and the business costs will be steered by the parameter δ.

$$x(x_P, x_G) = \delta(x_P + \theta x_G(x_P, d_{iv}))$$

The Grid Module.

The Grid Module is important for the calculation of the business costs for the existing infrastructure C_{GC} which come into play if net segments have to be reinforced due to load flow changes because of newly installed plants. New RES capacity can be the reason for the development of bottle necks in the grid system, through the generation of a high amount of electricity, which may be larger than the actual transportable capacity of the existing grid.

For illustrating this phenomenon it is necessary to develop a simplified network approach which contains basic physical regularities. Nolden et al. (2013) provide an overview on the common models used for techno-economic approaches. DC-Models with various extensions dominate the examination especially for policy investigations (Schweppe et al. 1987; Weigt 2006; Leuthold et al. 2008). Because of excluding network losses, reactive power and phase angles differences DC-Models make it possible to shape the load flow problematic in a linear way. If the impedance is known and the phase angles are constant, the equation of the Ohm's law shrinks $I = 1 / R$. In the case of our model R is defined as distance between two connection grid points.

The edge between two neighbour connection points v and $v+1$ is defined as e. For the calculation of the load flow change within a net segment e, the feed in electricity amount v is decisive.

$$PTDF_{ve} = \frac{\Delta y_e}{\Delta y_v}$$

The Power Transfer Distribution Factor (PTDF) (cf. Duthaler 2007) has to be calculated for all network segments and connection gird points. As a result a PTDF-Matrix is defined. Therefore it is possible to calculate the load flow by the summarization of all electricity feed-ins of all connection point.

Consumer centres with their associated demand are defined as load points. Via a composition assumption the generation (electricity production of RES plants) form any connection point has to be assigned to the different load points. The ratio between the total demand and the demand of a consumer centre is applied.

$$Y_{jv} = \frac{{Y^D}_j}{\sum_{j=1}^{J} {Y^D}_j} Y_v$$

Afterwards all supplies of electrical power at any connection point to all load points for every net segment are summarized. As a result we get the load flow capacity for all net segments at the time t.

$$cap_e^t = \sum_{j=1}^{J} \sum_{v=1}^{V} P_{jv} PTDF_{\ ve}$$

If the starting capacity of the net segments at $t = 0$ is exceeded, a reinforcement of the particular net segment will be necessary to guarantee supply security. The level of the capacity exceedance is multiplied by the factor price w_{GE} and therefore, determines the business costs for the existing net segment ${C_{GE}}_e$. After summarizing all arising costs for the net segments we get the total business costs for the existing grid system, which have to be paid by the consumer centres dependent on the ratio between the total demand and the demand of the particular consumer centre.

$$C_{GE} = \sum_{e=1}^{E} C_{GE_e} = \begin{cases} w_{GE}\ (cap_e^t\ - cap_e^{t=0}) & if \quad cap_e^t > cap_e^{t=0} \\ 0 & else \end{cases}$$