In the short-run we are all dead: Non-Equilibrium Dynamics in a Computational General Equilibrium model.

Davoud Taghawi-Nejad

Abstract. Studies of the economic impact and mitigation of climate change usually use computable general equilibrium models (CGE). Equilibrium models, as the name suggests, model the economy as in equilibrium, the transitions to the equilibrium are ignored. In the time spend outside equilibrium, the economy produces different quantities of goods and pollution as predicted by the equilibrium model. If the economy in this time outside of the equilibrium produces a different amount of climate gasses the predictions could be dangerously wrong.

We present in this paper a computational generalization of the Arrow-Debreu general equilibrium model, which is not in equilibrium during the transitions, but converges to the same equilibrium as a CGE model with the same data and assumption. We call this new class of models Computational Complete Economy models.

Computational Complete Economy models have other interesting applications for example in international trade, tax policy and macroeconomics.

Keywords: GCE, Climate Change, International Trade

1 Introduction

Studies of the economic impact and mitigation of climate change usually use computable general equilibrium models. Equilibrium models, as the name suggests, model the economy as in equilibrium; the out-of-equilibrium transitions to the equilibrium are ignored. In the time spend outside equilibrium, the economy produces different quantities of goods and pollution as predicted by the equilibrium model. If the economy in this time outside of the equilibrium produces more climate gasses the predictions are dangerously wrong.

We present in this paper a computational generalization of the Arrow-Debreu general equilibrium that is not in equilibrium during the transitions, but converges to the same equilibrium as a CGE model with the same data and assumptions about production and consumption functions.

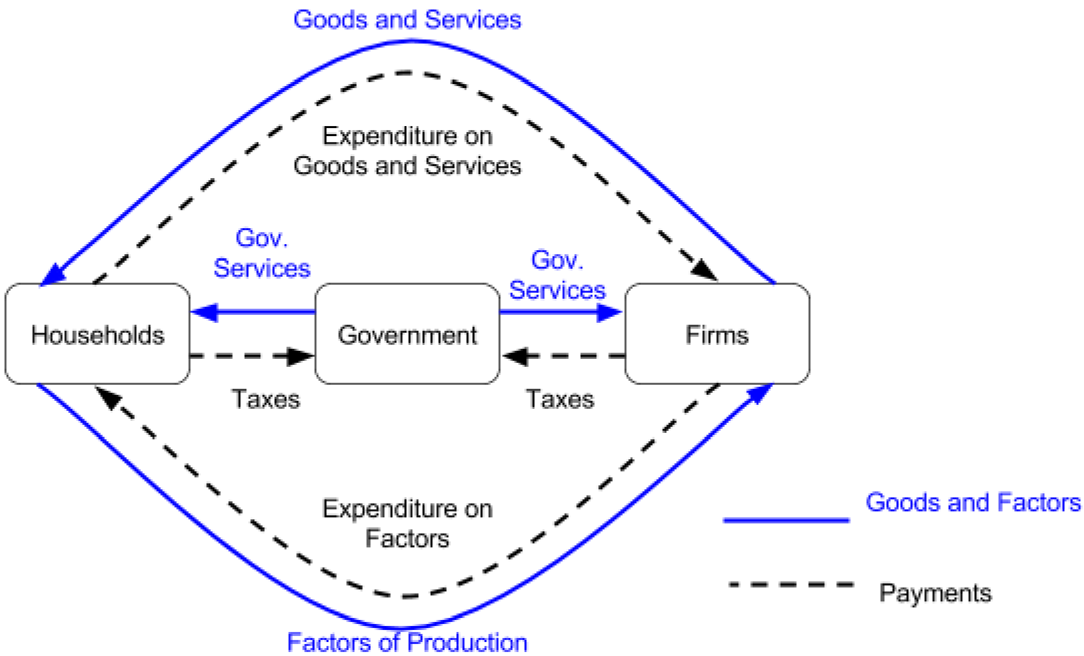
The aim of this paper is to demonstrate how to transform a Computable General Equilibrium model into a "Computational Complete Economy" (CCE) model - an agent-based model with transitions. In order to achieve this we use a widely cited CGE model and recreate in as a CCE model. The emphasis here is not only on the model itself but also on the strategy to calibrate the model.

In section two we will recapture the underlying CGE model. In section three we will explain how a CGE model is calibrated. In section four we will explain how the CCE model works and converges to equilibrium. In section five we will run policy experiments we will compare the asymptotic results with the CGE model and discuss the transitory effects of the experiment.

2 The underlying CGE model - producer, consumer and government behavior

2.1 Walrasian Equilibrium and the Circular flow of the economy

For many of the readers the circular flow model of the economy should be familiar.



The circular flow of the economy shows the flows of products and factors and their counter transaction, the payment for these goods and products. Households supplies factors of production - capital and labor - to the firms, which in turn supply goods and services. In the counter direction money flows from the household to the firms as a payment for the goods and services. The firm in turn pays the household for its factor provision (profit and factor income). The government collects taxes and provides government services.

The material flows must be balanced in the circular flow, that means that every factor provided by the household must be used by the firm and every good and service produced must reach the consumer. In equilibrium the value of the goods and services must balance the value of the factors. Otherwise value would just appear out of thin air. That implies also that the payments for factors balance the payments for goods and services. In other words the markets clear and there are zero economic profits.

Owed to computational restrictions and data availability CGE models typically assume representative agents. In the following we will assume that there is one representative household and on representative firm for each sector. We will also later on introduce a government agent, which only redistributes and a net-exports agent that captures international trade.

The indices $i = \{1, ..., N\}$ denote the set of commodities, $j = \{1, ..., N\}$ the set of industry sectors, $d = \{1, ..., D\}$ the set of final demands, and $f = \{1, \ldots, F\}$ represents the factor inputs - capital and labor. The circular flow in this economy can be completely characterized by three data matrices: an $N \times N$ input-output matrix of industries' uses of commodities as intermediate inputs, denoted by X, an $F \times N$ matrix of primary factor inputs to industries, denoted by V, and an $N \times D$ matrix of commodity uses by final demand activities, denoted by G. Figure 1, taken from Wing [2] represents this social accounting matrix.

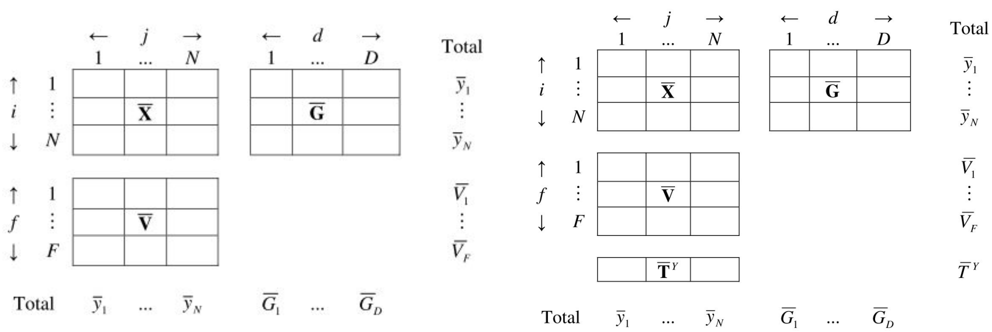


Figure 1 depicts the naming of entries, rows and columns of the SAM. The SAM is a matrix where each cell is the value of goods transferred from the row to the column entry. As it is custom in CGE models we normalize the quantity of goods in such a way that the prices in the SAM are 1. There are n firms their total value of output and in equilibrium also input is $\bar{y}_j$. The value of the input of a firm j is $\sum_i x_{ij} + \sum_f v_{fj}$ where x_{ij} are intermediate goods and v_{jf} are factors. The d consumers' consumption is g_{id} their total consumption is G_d. Consumers, can be the household's consumption, the household's investment, net-export and the government. V_f is the total provision of each factor by the consumers.

We will now illustrate how the social accounting matrix translates in equations for a circular flow:

First market clearing implies that the output of each sector must be equal to the input of all sectors plus the final use by the consumer for the commodity that sector produces:

$$\overline{y_i} = \sum_j x_{ij} + \sum_d g_{id} \tag{1}$$

Similarly the factor market must clear, each factor endowment must be equal to its use:

$$\overline{V}_f = \sum_i \overline{v}_{fi} \tag{2}$$

The zero profit condition implies that the gross output of each sector j is equal to the value of its input

$$\overline{y_i} = \sum_h x_{hi} + \sum_f v_{fi} \tag{3}$$

The income of the consumers, including government, investment and net-export - income from rental of primary factors, must be equal to the gross expenditure on commodity demands:

$$\overline{m} = \sum_f \overline{V_f} = \sum_i \sum_d \overline{g}_{id} \qquad (4)$$

2.2 The Cobb-Douglas Economy

In order to model the economy, we need to assume production and utility functions that characterize the economy. In this first description of a generalized arrow-debreu equilibrium we choose simple cobb-douglas economy. This choice is by no means necessary the algorithm works also for other functional forms such as the CES production function[1]:

The household maximizes its utility:

$$\max\ u(g_{1u}, ..., g_{iu}) = \prod_i g_{iu}^{\alpha_i} \qquad (5)$$

$$\text{subjected to} \quad \sum_i p_i(g_{iu} + g_{is}) = \sum_f p_f V_f\,.$$

where V is the factor endowment of the household. The factor endowment are the row sums of capital and labor is the SAM. g_{is} is the net saving, investment - net exports, which in this particular example is treated as fixed. The government is not modeled as a consumer. Analytically solving this problem leads to

$$g_{iu} = \frac{\alpha_i}{p_i}\left(\sum_f p_f V_f - \sum_i p_i g_{is}\right) \qquad (5a)$$

By solving for α, we are able to find the exponents of the cobb-douglas equation :

$$\alpha_i = \frac{g_{iu} p_i}{\left(\sum_f p_f V_f - \sum_i p_i g_{is}\right)} \qquad (5b)$$

$p_i^x g_{iu}$ is the value of good or factor i delivered to the household. We can read it from the SAM. It is the value in the row of good or factor I and the household column. $\sum_f p_f V_f$ on the other hand is income of the household, $\sum_i p_i g_{is}$ are the net saving, which can be read from the SAM, the former by adding the sum of the capital and labor columns; the later by summing up the columns for investment and net exports.
The assumption that the household is utility maximizing allowed us to infer the parameters of the utility function.

The firms' production functions

In our model each producer maximizes profit:

$$max\ \pi_j = p_j y_j - \sum_i p_i x_{ij} - \sum_f p_f v_{fj} \qquad (6)$$

$$\text{subjected to } y_j\left(x_{1j},\ ...,\ x_{ij}\right) = b_j \prod_i x_{ij}^{\beta_{ij}} \prod_f v_{fj}^{\beta_{fj}}$$

[1] For an application of the same algorithm in a CES production economy see [1] and [7]

solving this leads the following demand functions [2]

$$x_{ij} = \beta_{ij}\frac{p_j y_j}{p_i} \tag{7a}$$

$$v_{fj} = \beta_{fj}\frac{p_j y_j}{p_i} \tag{8a}$$

Rearranging this yields to

$$\beta_{ij} = \frac{p_i x_{ij}}{p_j y_j} \tag{7b}$$

$$\beta_{fj} = \frac{p_f x_{fj}}{p_j y_j} \tag{8b}$$

As in the household maximization we can infer the productions functions parameters. The denominator is the value of the production of firm j. It can be infer from the sum of the row entries of firm j or, equivalently, from the sum of the column entries of firm j. The numerator is the value of input of good i or factor f in the production of firm j. The value can be inferred from the corresponding (i, j) or (f, j) cell.

Until now we left out tax treatment. The social accounting matrix in the appendix includes taxes on the output of each firm. That implies that the expenses of each firm are distributed on input and on taxes, which lets us reformulate (7b) and (8b) to:

$$\beta_{ij} = \frac{p_i x_{ij}}{(1-\tau_j) p_j y_j} \tag{7c}$$

$$\beta_{fj} = \frac{p_f x_{fj}}{(1-\tau_j) p_j y_j} \tag{8c}$$

2.3 General Equilibrium

Given the equations 1 - 4 and the equations 5 - 7, we can now formulate our equilibrium system:

$$\overline{y_i} = \beta_{ij} p_j y_j + g_{iu} + g_{is} \tag{9}$$

Where g_{is} is the sum of all columns, other than g_{iu}, in submatrix G of the SAM.

The factor use remains unaffected

$$\overline{V}_f = \sum_i \overline{v}_{fi} \tag{10}$$

Assuming zero profits equation 6 implies:

$$p_j y_j = \sum_i p_i x_{ij} - \sum_f p_f v_{fj} \tag{11}$$

The value of output generated by producer j must equal the sum of the values of the inputs of the i intermediate goods and f primary factors employed in production.

Finally the agents' expenditure must be equal to their income:

[2] for details see [3]

$$\overline{m} = \sum_i \sum_d g_{id} = \sum_f w_f V_f \tag{12}$$

Substituting 5a and 7a into 9, we retrieve the excess demand vector for goods:

$$\Delta_i^C = \sum_j \beta_{ij} p_j y_j + \alpha_i \left(\sum_f p_f V_f - \sum_j p_j g_{js} \right) + p_i g_{is} - p_i y_i \text{ for all } i \in \{1, \ldots, N\}$$

And by substituting 8a into equation 10, we retrieve the excess demand function for the factor market

$$\Delta_f^F = \sum_j \gamma_{fj} \frac{p_j y_j}{w_f} - V_f \text{ for all } f \in \{1, \ldots, F\}$$

In general equilibrium the two excess demand vectors are minimized to zero.

The zero profit condition implies that producers profits are minimized to zero. Substituting 7a and 8a into the producer's profit function and dividing the production by cost, we can specify the excess profit vector:

$$\Delta_j^\pi = p_j b_j \prod_i \left(\frac{\beta_{ij}}{p_i} \right)^{\beta_{ij}} \prod_i \left(\frac{\beta_{fj}}{w_f} \right)^{\beta_{fj}} - \sum_i \beta_{ij}$$

Finally we define the excess income over agent return from the endowment:

$$\Delta^m = \sum_f w_f V_f$$

The general equilibrium is the joint minimization of $\Delta^C, \Delta^F, \Delta^\pi, \Delta^m$. If we stack the four vectors we have a system of 2N+F equations, Δ^* with 2N+F+1 unknowns, comprising the activity vector y with N variables, the commodity prices p with N variables, primary factor prices w with F variables and the scalar income level m.

The problem of finding general equilibrium is no finding $\Delta^*(z^*) = 0$, where z is the stacked vector of the unknowns - y, p, w and m. In order to have an economically meaningful general equilibrium we need to stipulate the all quantities in the vector z are non-negative.

2.4 Equilibrium

According to Varian [5] there are two properties our general equilibrium system must obey to ensure the uniqueness of the general equilibrium. First the household must satisfy weak axiom of revealed preference - households need a stable preference ordering space of all possible prices and income levels. Secondly, the aggregate demand for any commodity or factor is non-decreasing in the prices of all other goods and factors (gross substitutability).

Mas-Colell [6] proves that constant elasticity of substitution utility and production functions, whose elasticities of substitution are greater than or equal to one, have a unique equilibrium in the absence of taxes and other distortions. As a cobb-douglas economy is a CES economy with an elasticity of substitution of one this also holds here. Foster and Sonnenschein [7] and Hatta [8] on the other side find evidence that distortions can lead to multiple equilibria even in this setting. For a detailed treatment of taxes and distortions see also Kehoe [9]. In conclusion we can say that for our model existence and uniqueness is guaranteed until we introduce taxes or subsidies.

3 Numerical Calibration

In order to calibrate our model we first apply standard CGE trick. We define one unit of each good so that the price of each unit is one. With this definition each entry in the social accounting matrix represents both the value and the quantity of each good traded.
With this definition equations 5b, 7c and 8c reduce to the following calibration definitions:

$$\alpha_i = \frac{g_{iu}}{\left(\sum_f V_f - \sum_i g_{is}\right)} = \frac{g_{iu}}{\sum_h g_{hs}}$$

$$\beta_{ij} = \frac{x_{ij}}{(1-\tau_j)y_j} \qquad (13)$$

$$\beta_{fj} = \frac{x_{fj}}{(1-\tau_j)y_j} \qquad (14)$$

The cobb-douglas multiplier can be derived from the definition of y_j:

$$b_j = \frac{y_j}{\prod_i x_{ij}^{\beta_{ij}} \prod_f v_{fj}^{\beta_{fj}}} \qquad (15)$$

Where g_{iu} is the spending of the household on good i; V_f is the factor income of the household; g_{is} is the net saving, investment - net exports; x_{ij} is the the spending of firm j on good i; and y_j is the total output of or equivalently the total spending on sector j. All this values can be readily read from the input output matrix.

$$\tau_j = \frac{t_j}{(\sum_i x_{ij} + \sum_f x_{fj} + t_j)}$$

Where t_j is the tax paid by industry j on its output.

As we are holding net investment and net-export constant:

$$s_i = g_{is} \qquad (16)$$

And finally the factor endowments are the factor endowments from the SAM and the income of the household agent is its factor endowment:

$$V_f = V_f \qquad (17)$$

$$m = \sum_f V_f \qquad (18)$$

If we set the parameters of the model Δ^* according to the equations (14) - (18) based on the SAM, and solve $\Delta^*(z) = 0$, z replicates the SAM. In other words our model economy parameterized according to the SAM tends in equilibrium to replicate the circular flow of the economy described in the SAM.
However, as we will show in the next section and by numerical simulation the non-equilibrium computational complete economy model, this calibration does also lead to a CCE model that asymptotically replicates the SAM.

4 The computational generalisation - out of equilibrium transitions

In traditional CGE models we established our system of excess demand and profit functions Δ^* in order to reproduce the equilibrium result we just calibrated our functions and search, usually by non linear programming, for the z that satisfies $\Delta^*(z) = 0$ and $z \geq 0$.
In this work we are interested in transition to equilibrium and the transitions between equilibria. We therefore take our economy - equations (1) - (6) and calibrate them as described in section 2.4. With this description of the economy, we build a systemtem where time is modeled explicitly. Each time step firms trade goods at a price that assures market clearance and firms buying decision expressed in monetary terms is such that it corresponds to the excess profit function.
The following algorithm is hold generically it can be combined with different functional forms to replicate different CGE models. The strategy follows [1]:

Since we model time explicitly the simulation is a sequence of timesteps t. In each timestep the following happens:

1. Each agent sends the demand $\delta_{j,i} * w_j$,where δ is a weight that choosen to such that it maximizes $d_j(\delta_{j,i})$ and w is the wealth of the firm.

2. Each agent receives the nominal demand and given its current stock of output goods calculates the market clearing price:
$$\overline{p}_i^t = \frac{\sum_j^n \delta_{j,i} * w_j}{q_i^t}$$
when we assume that prices adapt with frictions:
$$p_i^t = \phi_p \overline{p}_i^t + (1 - \phi_p) p_i^{t-1}$$

3. If $\phi_p < 1$ and the system is not in equilibrium markets do not clear. The clients are therefore rationed proportionally to their demand. Excess supply is stored for next round. The resulting evolution of product stock is:
$$q_j^{t+1} = q_j^t - \overline{q}_j^t + f_j\left(\frac{\delta_{j,i}^t w_j^t}{p_i^t}\right)$$
where $\overline{q}_j^t$ is the amount actually sold:
$$\overline{q}_i^t = \frac{\sum_j^n \delta_{j,i} * w_j}{p_i^t}$$
if prices are fully flexible this reduces to:
$$q_j^{t+1} = f_j\left(\frac{\delta_{j,i}^t w_j^t}{p_i^t}\right)$$

4. We assume that the firms pay a λ dividend, its evolution of wealth is therefore:
$$w_i^{t+1} = (1 - \lambda)\overline{q}_i^t p_i^t$$
$$w_{hh}^{t+1} = q_{lab}^t p_{lab}^t + q_{cap}^t p_{cap}^t + \lambda \sum_i^n \overline{q}_i^t p_i^t$$
where hh is the household and cap and lab capital and labor

5. In order to calculate i's demand for input goods a firm maximizes the following function:
$$max\ d_j(\delta) = f_j\left(\frac{\delta_{j,0}}{p_0} \ldots \frac{\delta_{j,i}}{p_i} \ldots \frac{\delta_{j,n+2}}{p_{n+2}}\right)\ s.t.\ \sum_i^{n+2} \delta_i = 1 \qquad (1)$$

where p are last rounds prices. The resulting $\delta_{j,i}$'s are the share of the firm's capital w_j that are the demand ($\delta_{j,i} * w_j$) for input good i from firm i. i=n+1 and i=n+2 is the demand for capital and labor from the household. The exact functional form of $d_j(\delta_{j,i})$ is a result of production function $f_j(x_i)$. However the strategy for to maximize the function $d_j(\delta_{j,i})$, which is a transformation of $f_j(x_i)$,where x_i is divided by the price of x_i is universal to CCE models.
Also here we can assume that the adaptation is slow in this case:

$$\delta_i^{t+1} = \tau_\delta \overline{\delta}_i^t + (1 - \tau_W)\delta_i^t$$

where $\overline{\delta}$ is the solution of the maximization.
In this particular CCE:

$$max\ d_j(\delta) = b_j \prod_i \left(\frac{\delta_{ij}}{p_i}\right)_{ij}^{\beta_{ij}} \prod_f \left(\frac{\delta_{fj}}{p_f}\right)_{fj}^{\beta_{fj}}\ s.t.\ \sum_i^{n+2} \delta_i = 1$$

The two parameters that regulate the speed of adaptation in this model are ϕ and τ . The former regulates the speed of price changes the latter the speed of technological adaptation.

5 Simulation

The model has been implement the ABCE Agent-Based Computable Economy framework in python [5] and can be accessed online from http://52.90.210.1/.[3] In order to test the validity of the model we run the simulation with the taxes - τ_j - as implied by the calibration. The computational complete economy model must and does asymptotically reproduce the social accounting matrix. (Table 1 and 2 in the appendix) It is therefore asymptotically equivalent to a CGE model. We repeat this exercise with various output tax rates and compare the results with the outcomes of CGE model. It also here produces asymptotically the results CGE models produce as equilibrium results.
The most interesting application of this CCE model and the underlying CGE model is the introduction of a tax on carbon. For this we modify (6) to include a further carbon tax:

$$max\ \pi_j = (1 - \tau_j)p_j y_j - \tau^{carb} e_j y_j - \sum_i p_i x_{ij} - \sum_f p_f v_{fj} \tag{6}$$

$$\text{subjected to } y_j\left(x_{1j},\ ...,\ x_{ij}\right) = b_j \prod_i x_{ij}^{\beta_{ij}} \prod_f v_{fj}^{\beta_{fj}}$$

Where τ^{carb} is the carbon tax, e_j is the carbon emission in tons of CO_2 per unit of output y_j. It is important to note that $-\tau^{carb} e_j y_j$, does not enter the maximization in step 1 of the CCE algorithm, but it implicitly enters the algorithm by modifying the capital available for buying inputs.

The introduction of a tax on carbon produces equivalent results only in a limited range. From 0 to 82.4 dollars per ton of carbon, the result of the CCE model and the CGE model are asymptotically the same, but above this the CCE model starts to oscillate instead of converging to the CGE result. Interestingly the mean of the oscillation is still the same as the CGE result.

We regard this as a weakness of the underlying CGE model rather than the CCE model. It has been demonstrated in [6], [7] and [8], the introduction to tax distortion leads to non uniqueness of the equilibrium.

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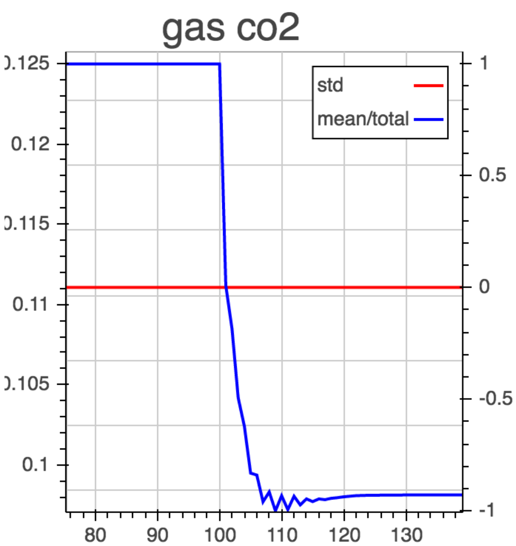
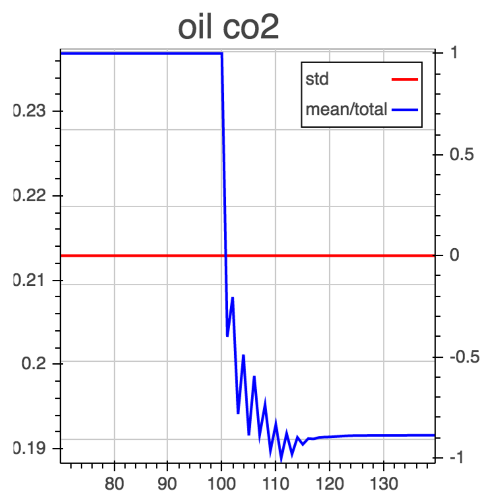
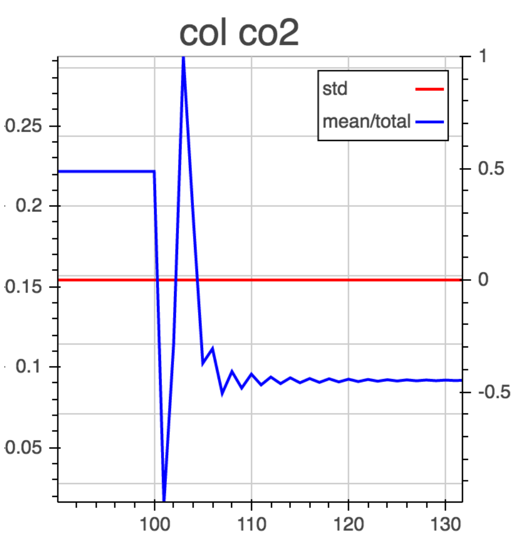
[3] It can be downloaded from
https://bitbucket.org/DavoudTaghawiNejad/cce-computational-complete-economy-model.git

Exemplary we look at a 50$ per ton carbon-dioxide tax. In this stage, we have not empirically calibrated the adaptation speed of wage, prices and the technology. To illustrate out of equilibrium transitions we assume intermediary values of 0.5 for each. Asymptotically the model produces the same emissions as a CGE model produces in equilibrium - the total CO_2 emissions decrease from 5834 million tons to 3814 million tons[4]. But the transitions tell a richer story. While the CO_2 emissions produced by the oil and the gas sector decrease, with fluctuations and temporarily produce less emissions than in equilibrium, the CO_2 production of the coal sector do initially increase and then approximate the equilibrium output.







[4] Which is approximately equal to the outcome of the CGE model, which predicts 5795 million tons without the tax intervention and 3814 with the tax intervention. (running the simulation for a longer time makes the approximation better)

6 Conclusion

We have build a model that gives the same asymptotic results as a CGE model. This model has the same constraints and limitations as a CGE model: one representative agent per sector and multiple equilibria in face of tax distortions. The model demonstrates that there are transitional paths before equilibrium. Depending on the speed of adaptation of the production technology, prices and wages the transitory output of CO_2 can be substantially different from the equilibrium result. The conclusion is that clearly CGE models are too simplistic and must be replaced to adequately assess environmental policy.

We have demonstrated that CGE calibration techniques can be used to calibrate an agent-based model and that in an ABM that structurally similar to a CGE model the results are asymptotically identical.

We do not propose to stop there. Agent-Based models have the potential to be much richer and allow to model more realistic assumptions than general equilibrium models. With this paper we hope to encourage researchers to build richer agent-based models that use CGE calibration techniques, but are not asymptotically equivalent.

7 Appendix

Table 1. The Social Accounting Matrix of the US in 2000, from [2] based on Bureau of Economic Analysis data files; in 10^{10} dollars

	col	ele	gas	o_g	oil	eis	trn	roe	lab	cap	hoh	inv	nx	tax	sum
col	0.243	1.448	0.004	0	0.001	0.219	0.013	0.238			0.014	0	0.108		2.288
ele	0.052	0.084	0.027	0.118	0.168	1.384	0.283	9.53			12.915	0	-0.093		24.468
gas	0.003	0.526	2.283	0.446	0.246	0.817	0.056	2.199			4.136	0	0.045		10.757
o_g	0	0.024	4.795	2.675	8.381	0.939	0.03	0.12			0.013	0.072	-6.189		10.860
oil	0.066	0.238	0.038	0.072	1.753	0.628	2.428	4.95			8.345	0.128	-0.542		18.104
eis	0.101	0.121	0.015	0.285	0.513	17.434	0.177	47.534			9.239	0.906	-3.506		72.819
trn	0.158	0.945	0.135	0.122	0.784	3.548	9.796	19.835			17.316	1.492	5.107		59.238
roe	0.747	5.142	1.897	4.694	2.798	19.974	16.055	540.977			751.254	203.063	-21.41		1525.191
lab	0.437	4.422	0.434	0.665	1.141	16.128	19.032	553.948							596.207
cap	0.278	8.83	0.866	1.525	2.115	10.806	9.792	310.641							344.853
hoh									596.207	344.853			26.476	41.357	1008.893
inv											205.661				205.661
nx															0.000
tax	0.203	2.686	0.263	0.258	0.204	0.944	1.574	35.225							41.357
sum	2.288	24.466	10.757	10.860	18.104	72.821	59.236	1525.197	596.207	344.853	1008.893	205.661	-0.004	41.357	

Table 2. Simulation results without policy change

	col	ele	gas	o_g	oil	eis	trn	roe	household	Invest. + netexport	sum
col	0.243	1.448	0.004	0	0.001	0.219	0.013	0.238	0.014	0.108	2.288
ele	0.052	0.084	0.027	0.118	0.168	1.384	0.283	9.528	12.915	-0.093	24.466
gas	0.003	0.526	2.283	0.446	0.246	0.817	0.056	2.198	4.136	0.045	10.755
o_g	0	0.024	4.793	2.674	8.38	0.939	0.03	0.121	0.013	-6.117	10.858

oil	0.066	0.238	0.038	0.072	1.753	0.628	2.428	4.949	8.345	-0.414	18.102
eis	0.101	0.121	0.015	0.285	0.513	17.437	0.177	47.533	9.239	-2.6	72.82
trn	0.158	0.945	0.135	0.122	0.784	3.548	9.796	19.834	17.316	6.599	59.236
roe	0.747	5.143	1.897	4.694	2.798	19.973	16.055	540.98	751.252	181.657	1525.195
Household (=lab + cap)	0.715	13.252	1.3	2.189	3.256	26.931	28.824	864.591	0	0	941.058

Table 3. Impacts of a 50$ CO_2 tax - CGE model

	col	o_g	gas	oil	ele	eis	trn	roe
price impacts (percent change)	144.27	1.62	21.06	20.97	7	1.27	1.17	0.32
quantity impacts (percent change)	-58.63	-29.51	-21.47	-19.11	-6.58	-1.44	-1.28	-0.11
consumption impacts (percent change)	-58.84	-1.05	-16.94	-16.88	-6.03	-0.71	-0.62	0.23
coal input impacts by sector (percent change)	-81		-66.17	-65.85	-59.08	-59.14	-59.11	-58.97
oil input impacts by sector (percent change)	-61.64	-40.79	-31.69	-31.04	-17.37	-17.49	-17.44	-17.16
gas input impacts by sector (percent change)	-61.67	-40.83	-31.74	-31.08	-17.43	-17.55	-17.5	-17.22
electricity input impacts by sector (percent change)	-56.63	-33.06	-22.78	-22.03	-6.58	-6.72	-6.66	-6.34

Table 4. Impacts of a 50$ CO_2 tax - CCE model (asymptotical)

	col	o_g	gas	oil	ele	eis	trn	roe
price impacts (percent change)	144.28	1.62	21.06	20.98	7	1.28	1.19	0.34
quantity impacts (percent change)	-58.62	-29.49	-21.46	-19.1	-6.58	-1.42	-1.29	-0.11
consumption impacts (percent change)	-58.84	-1.07	-16.96	-16.9	-6.05	-0.74	-0.65	0.2
coal input impacts by sector (percent change)	-81	0	-66.16	-65.8	-59.07	-59.12	-59.1	-58.96
oil input impacts by sector (percent change)	-61.63	-40.77	-31.69	-31.03	-17.36	-17.46	-17.41	-17.14
gas input impacts by sector (percent change)	-61.65	-40.81	-31.74	-31.08	-17.42	-17.51	-17.47	-17.19
electricity input impacts by sector (percent change)	-56.62	-33.04	-22.77	-22.02	-6.57	-6.68	-6.63	-6.31

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