Demand Based Balance of Flow Manpower Modeling for Education Policy in Afghanistan

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Abstract. This research uses a demand based balance of flow manpower model premised in mathematical programming to provide insight into the potential futures of the Afghan Education System. We will later use this research to inform and help validate a large scale simulation of the same topic. Over the previous three decades, torn from multiple wars and an intolerant governing regime, the education system in Afghanistan has been decimated. Over the past 10 years Afghanistan and the international community have dedicated a substantial amount of resources to educate the Afghan youth. By forecasting student demand we are able to determine points of friction in the teacher production policy regarding grade level, gender, and province across a medium-term time horizon. Future work will entail alternative scenarios including shortened length of teacher training, relaxation of geographic restrictions, and stochastic analysis of the attrition model.

Keywords: Afghan education, education policy, manpower model, mixed integer linear program

1 Introduction

Over the previous three decades, torn from multiple wars and an intolerant governing regime the education system in Afghanistan has been decimated. Only in the recent decade has there been unified effort toward the improvement of education. This emphasis regarding education has provided benefit, but has also brought unexpected problems. There has been a seven fold increase in the demand for primary and secondary education with nearly seven million children enrolled in school today [10]. Unfortunately, in a country with 27% adult literacy, an ongoing war upon its soil, an opium trade as its primary gross domestic product, and an inefficient use of international aid — meeting the increasing demand for education is difficult at best [12].

As of 2012, there are 56 primary donors who have donated approximately $57 billion U.S. to Afghanistan [8]. In 2014, with the anticipated withdrawal of coalition forces and a newly autonomous Afghan state, the future is uncertain. The purpose of this research is to use mathematical modeling to demonstrate potential outcomes and points of friction regarding the demand for teachers in Afghanistan given the substantial forthcoming changes in the country. This research will eventually support the development and validation of a complex simulation meant to inform policy decisions regarding education in fragile states.

"Teacher management is a critical governance issue in fragile state contexts, and especially those in which the education system has been destroyed by years of conflict and instability" [7]. For this reason, this project focuses on the Afghan government's capacity for teacher training as it pertains to the growing demand for education. Although the current pool of teachers has a mixed training background (73% of teachers have not met the grade 14 graduate requirement [2]), the Afghanistan Ministry of Education requires a two year teacher training college (TTC) after a potential teacher has passed the equivalent of 12th grade [10]. Therefore, it is rather important to determine the annual requirement for teachers entering the training base. Of equal importance is discovering potential weaknesses in training capacity, and where these potential friction points exist. The issues cannot be remedied in the short run; therefore, it is beneficial to use insights gained through modeling to inform policy decision preventing the exacerbation of shortages in the out years.

This work presents a network flow linear program that provides insight into potential weaknesses in the development of the future of the Afghan education system. As with most manpower models, the purpose is to provide decision makers insight to ensure training and inventory focus on the areas needed [4]. We use Bombach's individual demand approach to education forecasting, where "the potential supply of educated manpower is derived from the present and the expected future individual demand for education. The projection is based on the rate of growth and the age composition of population, the present structure of the educational system, the number of students already enrolled, the prevailing graduation rates and trends, and the possible changes in the social structure of inflow into education" [3]. The proposed model determines the number of teachers that should be in training across Afghanistan by province, grade, gender and time. The change in requirements by year is based on an increasing demand for education due to population increase, desire, near spontaneous development, and parity across gender and social class. The model's primary premise is that of a balance of flow [3]. The number of teachers you can have in any given year is dependent on the number you had the previous year, the number that stop teaching (quitters, deaths, retirees), the number that complete training, and the number that are reassigned within elementary, middle, and high school. Unfortunately, due to a fledgling current teacher base and limited teacher production capability, meeting the desired student teacher ratio in every province is infeasible, therefore we developed an integer program with elastic constraints to allow for a feasible solution set.

2 Data and Assumptions

Below we explain the origins of the education data, development of population data, and the key assumptions used in this research.

The MOE uses the Educational Management Information System (EMIS) to collect a substantial amount of data regarding its teachers and students, providing transparency to the people of Afghanistan, as well as the international donor community. We feel this data, although not 100% accurate, provides enough truth to allow modeling.

Due to the difficult situation in Afghanistan, we extended the current United Nations charter for 100% enrollment in primary education by 2015 to 2025. The intent of the model is to provide insight at 2020. We use truncation to mitigate end effects regarding this relatively short horizon [6]. Therefore, we terminate the model in the year 2025.

The UNESCO Institute for Statistics (UIS) projections estimate the number of additional teachers, or inflow, required to compensate for attrition rates in order to assess potential training needs. Based on UIS data, three scenarios are developed according to the attrition rates: low (5%), medium (6.5%) and high (8%) [13].

The current annual salary for a teacher is $2000 US (gross approximation due to a disparity across the country).

30 student classrooms provide the optimal size, and is therefore the goal by 2025 [1].

Although relaxed in some urban areas, gender separation is a requirement in much of the country, and the MOE is vying for gender parity. We therefore assume no cross gender education (except in TTC).

Teachers that attend TTC in a province will teach in that same province.

UN population estimates for 2025 were used [14]. [2] A suitable mathematical technique is a linear extrapolation of the population based on the futures of the ages and the current population proportion across provinces [9]. This method provides values relatively close to the MOE's provided in the 2011 Ministry of Education Interim Plan. Equation (1) shows the population growth model used, where y is the year, g is the grade, s is gender, p is province and ts is the year's ordinal value. *GrowthRate* is determined using the U.N. population forecast for Afghanistan each year and assuming the 100% of primary aged school children and 80% of secondary aged school children are enrolled by 2025. This assumption was proportionally decomposed across province, sub-age group and gender.

$$StudentPopulation_{y,g,s,p} = StudentPopulation_{2011,g,s,p} + ts \times GrowthRate_{g,s,p} \tag{1}$$

[1] The current plan by the MOE is 35.6 person classrooms by 2015.

[2] This is an aggregate of all 35 provinces, so we used the ratio of the population by province in 2009 from the LandScan 2009 Global Population Database (The NATO standard for assessment in Afghanistan) and used linear forecasting to determine future enrollment based on the Enrollment Ratio Method, "which is calculated on the basis of past data, and is extrapolated into the future by applying a suitable mathematical technique or a specific logic" [9].

3 Notation and Problem Formulation

Below is the problem formulation. The reader is asked to look at [5] as it is the source we used as a handrail for principals of education modeling.

The premise of the model is that of an employee hiring-training problem [15]. The MOE must hire, train, and graduate enough teachers from TTC each year to meet the growing demand of education in Afghanistan. The generic formulation of this manpower modeling can be stated as:

$$\text{Min} \sum_{t,d,bg,g} c(x_{t,d,bg,g}) + \sum_{t,d,bg,g} tc(y_{t,d,bg,g}) + \sum_{t,d,bg,g} penalty(slack_{t,d}) \tag{2}$$

Subject to

$$\sum_{g} x_{t,d,bg,g} = \sum_{g} x_{t-1,d,bg,g} - \sum_{g} losses_{t-1,d,bg,g} + \sum_{g} y_{t-2,d,bg,g} \quad \forall t,d,bg \tag{3}$$

$$\sum_{Z} x_{t,d,bg,g} = \sum_{Z} x_{t-1,d,bg,g} - \sum_{Z} losses_{t-1,d,bg,g} + \sum_{Z} y_{t-2,d,bg,g} \quad \forall t,d,bg \tag{4}$$

$$losses_{t,d,bg,g} \geq= \nu\% \times x_{t,d,bg,g} \tag{5}$$

$$u = st_{2012} - Ordinality(t)\left(\frac{st_{2012} - st_{2025}}{number\ of\ time\ steps}\right) \tag{6}$$

$$x_{t,d,bg,g} \geq \frac{r_{t,d,bg,g}}{u} \tag{7}$$

$$\sum_{g,bg} y_{t,d,bg,g} + \sum_{g,bg} y_{t-1,d,bg,g} \leq k_d(1+.1)^{ordinality(t)} + slack_{t,d} \qquad \forall\ t,d \tag{8}$$

$$x_{t,d,bg,g} \geq 0 \tag{9}$$

$$y_{t,d,bg,g} \geq 0 \tag{10}$$

where

$s_{t,d,bg,g}$	number of teachers at the starting time (time, province, gender, grade).
$r_{t,d,bg,g}$	projected number of students (time,province,gender,grade).
$k_{d,t}$	teacher training capacity by year and district (province, time).
c	scalar annual cost of a teacher.
tc	scalar annual cost of training a teacher.
u	scalar of students to teacher ratio.

Here $x_{t,d,bg,g}$ is the decision variable representing the number of employed teachers in year t, province d, of gender bg, and grade g, while $y_{t,d,bg,g}$ is the decision variable for the number of teachers in training across year t, province d, of gender bg, and grade g. $losses_{t,d,bg,g}$ indicates the number of teacher losses across parameters, both by expected attrition rates and additional releases based on population change. The model's objective is to minimize cost in dollars based on the annual salary of an Afghan teacher and the annual cost of training an Afghan teacher. Also, due to the infeasibility of meeting some of the constraints under variable conditions, elastic variables are used and the penalty is incorporated in the objective function. Constraint (3) bounds the primary flow for the entire system. This equation ensures that the number of teachers required in each year is equal to the number of teachers from the previous year subtracting the number of losses and adding the number of TTC graduates. Constraint (4) allows the model to move teachers within education levels of primary, secondary, and upper secondary exclusively. Z is a subset of g either pr, se, hs. This provides a more realistic hiring plan allowing provinces to shift teachers accordingly to fill gaps. Constraint (5) allows the model to plan losses, giving the flexibility to release teachers while maintaining the minimum losses by grade as forecasted by data (ν is the expected proportion of losses). Equation (6) and Constraint (7) constrain each province by grade and gender to meet a function of student teacher ratio. Here st is the student teacher ratio. The function is a linear model which begins in 2012 at the provinces current ratio and steps to the goal class size of 30 students per classroom by 2025. Constraint (8) is the capacity constraint for teacher training. Here a slack variable is used to account for a lack of capacity, providing elasticity and allowing the model to handle infeasibility. This slack variable accounts for the additional required capacity in the province's training base. This capacity corresponds to the required number of training positions in TTC for the respective province beyond that of current or forecasted capacity. Capacity is assumed to grow at 10% compounded annually. This is a bit lower than the MOE goal for the next three years, but is an adequate fit for the out years.

4 Results

The model was solved using the CPLEX solver as a Mixed Integer Linear Program in the General Algebraic Modeling Interface. The model had approximately 86,000 variables.

Based on current production estimates, reaching the goal of 30 student classrooms in the next thirteen years at the prescribed growth rate is infeasible in over half the provinces. However, the model provides enlightening insights into the requirements and why the infeasibility exists.

With a goal of converging from current classroom sizes to 30 person classrooms by 2025 and a TTC capacity growth rate of 10% compounded annually and expected losses of at least 6.5% for each gender respectively, there is a requirement to hire and train approximately 248,000 male teachers and 255,000 female teachers for a total of about 503,000 teachers over the next 12 years. On average this would cost approximately $571 million U.S. a year in teacher salaries alone (approximately 9% of Afghanistan's 2009 total international aid [11] and 30% of its 2010 annual revenues [1]).

In order to grow the teaching force to the required size while accounting for planned losses the current production based on the 10% growth will fall short by approximately 193,000 teachers in 18 of 35 provinces. Figure 1 shows the breakdown of provinces which will be short teachers in at least one year from 2013 to 2022. There appears to be geographic implications (which spatially correlate to social boundaries), some of which are population driven, some are due to urban and rural disparity, and some are socially driven (e.g. a lack of female teachers).

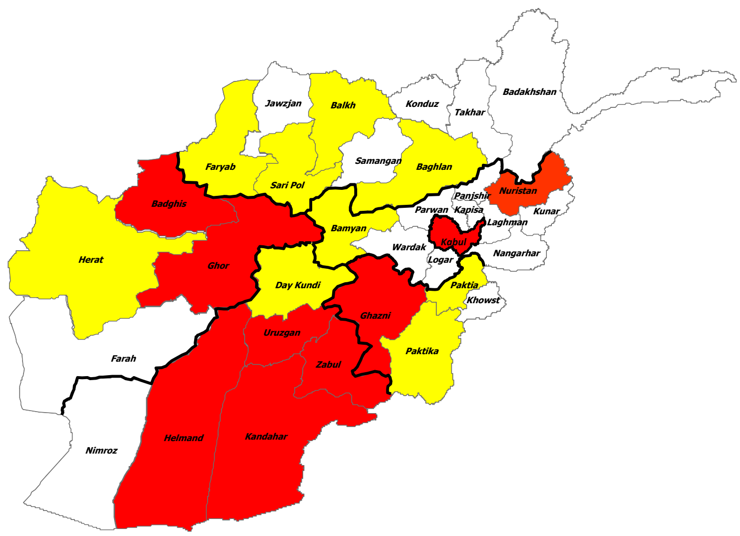

Fig. 1. Provinces Lacking Required Capacity. The colors represent the ratio of additional capacity required to teacher requirements. White provinces require no additional capacity, yellow provinces require some additional capacity, and red provinces require the greatest additional capacity.

Figure 2 shows the required employment of teachers across the education levels to meet the 2025 education requirements. We see that the initial focus will be amongst primary school teachers as the younger student population continues to grow at new found rates; however, by 2020 we begin to see increased growth of demand for secondary education, and by 2023 the upper secondary education growth rate becomes exceptional.

As of 2010, 245 out of 412 urban and rural districts did not have a single qualified female teacher – illustrating the gender disparity regarding teachers [2]. However, the population of students is growing to parity amongst genders [10]. We found that in the year 2020 the rate at which the model graduates and employs female teachers increases substantially above that of the men. Figure 3 shows the required convergence assuming that females must be taught by females. This is both based on the requirement for female teachers to teach females and the growing population of female students. Although this growth is infeasible due to capacity constraints, it is a basis to determine the required

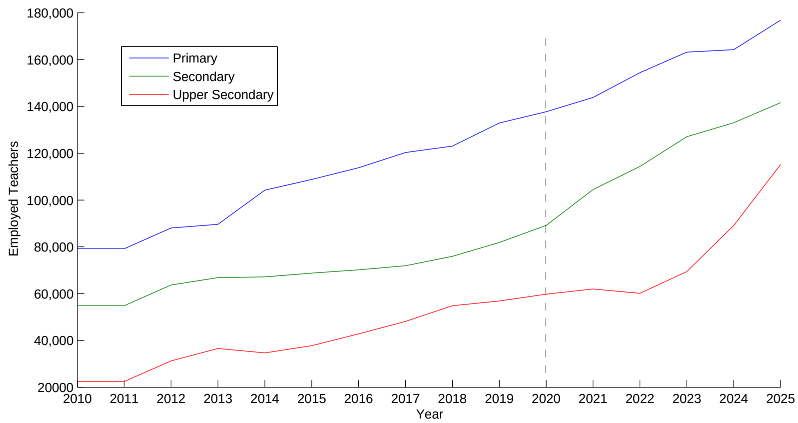

Fig. 2. Required Aggregate Teachers by Education Level Across Afghanistan to Meet 30:1 ratio in 2025.

growth in female teachers. It is likely infeasible to even provide this many 12th grade graduate females by 2025.

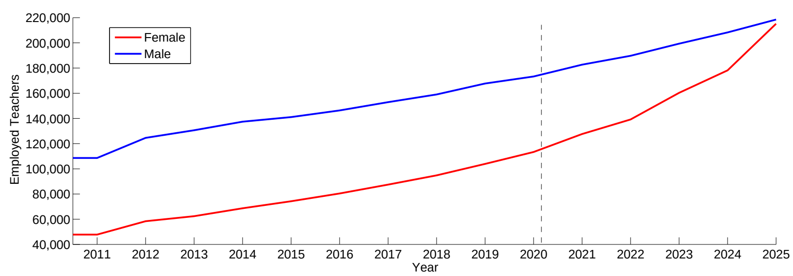

Fig. 3. Required Teachers employed by gender across all of Afghanistan.

Referring back to Figure 1, it became of interest to use the model to conduct further analysis into the provinces which lack the required capacity to meet demand. Figure 4 shows the 10 provinces with the greatest shortage in TTC capacity. Immediately, we notice, Kandahar, Ghazni, and Kabul (province) are consistently suffering from a lack of capacity. Of note, most of the distressed provinces display a rather stable growth in the required training capacity past 2014 with the exception of Ghor Province (black). Further investigation reveals that Ghor's overage and required growth in the 2022 to 2023 time frame is dependent on its current and excessively high female student to teacher ratio. Because of this – as the model moves to close this gap over the 12 year time span, it is at a 60:1 ratio in 2024 and moves to a 30:1 in 2025. Therefore, Ghor must double the number of female teachers in the province. This highlights an important point – the provinces are different, and it is going to take individual policies to grow the teacher population accordingly. In the case of Ghor, a linear growth rate is not feasible, and a greater rate of growth up front would be beneficial.

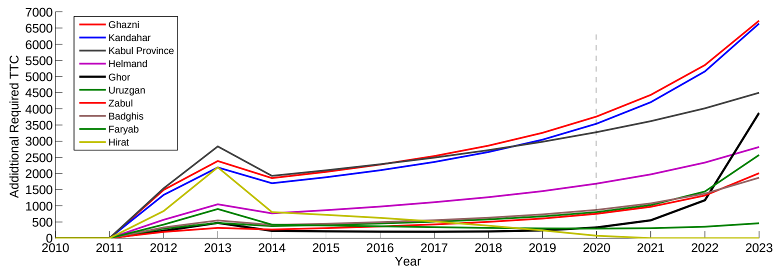

Fig. 4. Required Teachers employed by gender across all of Afghanistan.

Because the model accounts for region and gender, we are able to glean insights into the gender disparity in those provinces that display some of the greatest difficulty in meeting the requirements. For example, Kandahar and Ghanzi, display substantial disparity in the hiring of females in the early years to compensate for their current disproportion. However, in the mid years the model reaches equilibrium in hiring only to surge growth regarding female teachers in 2022. This correlates to what was seen in Figure 2 regarding a significant growth of upper secondary demand. In 2023 the model hires approximately 4,000 teachers for upper secondary in Ghanzi and 2,000 teachers for upper secondary in Kandahar, the proportion of these which are female is 76% and 100% respectively. Although this can be gleaned by conducting data analysis, we feel this is a successful instance where optimization can provide insights parsimoniously. The bottom line seems glaringly obvious, there is a lack of capacity to which there is likely no answer and this will likely cause disparity in education throughout the country unless other techniques for training and hiring teachers is used.

5 Conclusion

This paper presented a way to use a rather common linear programming technique to gain insight into a large, messy problem which has the international community's attention. The original intent and the true purpose of the model is as a validation tool for a following simulation; however, we believe the model will provide much more. Developed with minimum manpower and using open source data, the model employed a mixed integer linear programming premised on balance of flow to show provincial difficulties in meeting demand for the quickly growing and very important Afghan education system. Simultaneously, elastic constraints and a researched understanding of the problem provide insightful analysis regarding a rather unknown environment. We have been able to isolate those provinces in Afghanistan which are critically short either teachers or training capacity and clearly demonstrate the severity. This is by no means the end of the analysis. Using this model alone, we could sit side by side with the appropriate decision maker and delve deep into each province, district, grade or gender to provide an almost unlimited amount of futures analysis. Planned future analysis will provide insights into potential solutions and their impacts. Such solutions are shortening the requirement for TTC, relaxing geographic constraints regarding the relationship between TTC and actual employment, or ensuring adequate retention of teachers through stochastic optimization.

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6 ODD Protocol

The ODD protocol was developed primarily for describing IBMs and ABMs [3]. However, we feel the ODD protocol provides a good format to provide greater fidelity into most modeling paradigms — mathematical programming included. As much of the main paper meets many of the goals of the ODD, we use this space to delve deeper into some of the model's nuances and to describe the model in a framework which is becoming well understood to many practitioners of social simulation and modeling.

6.1 Purpose

The purpose of this research and the subsequent model is twofold. First, the model was initially built to provide a validation handrail for a larger, more complex simulation of the Afghanistan Education System. In modeling the education system, the model's output allows for investigation of the relationship regarding the supply and demand of teachers and the ability of the education system to create teachers to meet the growing demand for education.

6.2 State variables and scales

There are three classes of individuals represented in the model — *students*, *teachers*, and *teacher training college students*. The model accounts for students from grades 1-12 at the individual level. Students are assigned attributes of grade, province, and gender according to an annual indexing. Teachers have the same attributes; however, a teacher's grade corresponds to the grade he or she teaches in the current year. Teacher training college students are assigned a gender and province for the two years they remain in school.

There are three primary decision variables within the model. The model selects the required number of teachers by grade, province, and gender each year to meet the dynamic and required student to teacher ratio (demand) while minimizing cost. The model determines the number of teachers in training by province and year to meet the required ratio, while accounting for losses due to attrition or release of teachers. The model also has the ability to control the number of teacher losses. Although this is anchored by an annual attrition coefficient, the model is able to "fire" teachers under restrictions to allow flexibility to optimize production across provinces and time.

The below list explains the parameters, data, and decision variables. This provides an explanation of the model parameters and facilitates the use of this addendum as a stand alone document as needed.

Index

t	time periods(2011,2012...2025).
bg	gender (Male Female).

[3] Grimm et al. 2006

d	provinces (Kabul, Paktia).
g	grade(1..12) subsets are primary(1-4), secondary(4-8) and high school(9-12) .
Data	
$s_{t,d,bg,g}$	number of Teachers at the starting time (time, province, gender, grade).
$r_{t,d,bg,g}$	projected number of students (time,province,gender,grade).
$l_{t,d,bg,g}$	number of teachers leaving (time,province,gender,grade).
$k_{d,t}$	teacher Training capacity by year and district (province, time).
c	scalar annual cost of a teacher.
tc	scalar annual cost of training a teacher.
u	scalar number of teachers per student.
Decision Variables	
$x_{t,d,bg,g}$	number of teachers in classrooms.
$y_{t,d,bg,g}$	number of teachers in training.
$losses_{t,d,bg,g}$	number of teacher losses.
$slack$	we used elastic variables in order to allow relax constraints as any optimal goal is infeasible.

6.3 Process overview and scheduling

In this math programming model (as with most), there is no true time step as the model solves optimally across the required indexing mechanisms. However, we do use time by year as an index. This allows for annual estimates of both student population as well as teacher production. The student population model uses UN population estimates for 2025. This is an aggregate of all 35 provinces, so we used the ratio of the population by province in 2009 from the LandScan 2009 Global Population Database (The NATO standard for assessment in Afghanistan). We then used linear forecasting to determine future enrollment based on the Enrollment Ratio Method. In this case, a suitable mathematical technique is a linear extrapolation of the population based on the futures of the ages and the current population proportion across provinces. This method provides values relatively close to the MOE forecasts given in the 2011 Ministry of Education Interim Plan. Equation (11) shows the formula used.

$$StudentPopulation_{y,g,s,p} = StudentPopulation_{2011,g,s,p} + ts \times GrowthRate_{g,s,p} \tag{11}$$

The population is a primary driver for the demand constraint. The second driver for this constraint is the dynamic required student to teacher ratio.

$$u = st_{2012} - Ordinality(t)\left(\frac{st_{2012} - st_{2025}}{number\ of\ time\ steps}\right) \tag{12}$$

$$x_{t,d,bg,g} \geq \frac{r_{t,d,bg,g}}{u} \tag{13}$$

Where st is the teacher student ratio. Equation (12) and Constraint (13) constrain each province by grade and gender to meet a function of student teacher ratio. The function is a linear model which begins in 2012 at the provinces current ratio and steps to the goal class size of 30 students per classroom by 2025.

This allows for a linear movement from the provinces current student teacher ratio to the goal of 30:1 by 2025 in each grade and gender. We acknowledge in the main paper that the linear movement is not ideal, but it does prove suitable for most of the provinces.

The teacher production constraint which is discussed in the main paper is a standard exponential population growth formulation. The year 2011 production values were assumed maximum capacity for TTCs by province. These values are improved upon annually at a particular rate. For example, we used 10% for the analysis as it aligned well with the current estimations given by the MOE for the out years of 2012-2014. As shown in the main document:

$$\sum_{g,bg} y_{t,d,bg,g} + \sum_{g,bg} y_{t-1,d,bg,g} \leq k_d(1+.1)^{ordinality(t)} + slack_{t,d} \qquad \forall\ t,d \quad (14)$$

Constraint (14) is the capacity constraint for teacher training. Here a slack variable is used to account for a lack of capacity, providing elasticity and allowing model to solve. This slack variable accounts for the additional required capacity in the provinces training base. This capacity corresponds to the required number of training positions in TTC for the respective province beyond that of current or forecasted capacity. As stated, capacity is assumed to grow at 10% compounded annually. This is a bit lower than the MOE goal for the next three years, but is an adequate fit for the out years.

6.4 Design Concepts

Although the ODD design concepts do not directly relate to the optimization framework, we feel this is a good opportunity to explain a key aspect of the model. Clearly from the main paper we see that the goal of the model is to minimize the overall cost of education in Afghanistan while meeting demand related constraints. This is done through a balance of flow where the number of teachers you can have in any given year is dependent on the number you had the previous year, the number that stop teaching (quitters, deaths, retirees), the number that complete training, and the number that are reassigned within elementary, middle, and high school.

However, something that might not be as intuitive is the use of elastic constraints to allow for feasibility while accounting for shortages. Understanding that the goal for student teacher ratios within a district will likely be infeasible we developed elastic constraints. The elasticity allows teacher training colleges to train beyond their capacity, but at an extremely high cost which is captured in the objective function. The amount of additional capacity is accounted for across indexes providing qualitative insights into future frictions.

6.5 Initialization

The initial data for the model is based on the UN 2011 population estimates for Afghanistan decomposed across provinces via the LandScan 2009 Global Population Database. The initial number of students, teachers, and TTC students is based on the 2010 Afghanistan EMIS database which is open source data.

6.6 Inputs

Inputs are explained in the previous subsection as their explanation is required upfront. However, it is important that the reader understand that the linear extrapolation model used for population growth, the determined values for attrition coefficients, and the TTC capacity growth model all create data which is used by the model. Different assumptions which might create different functions will result in different model behavior.

6.7 Submodels

The following is a detailed model explanation.

Objective

$$\textbf{Minimize} \sum_{t,d,bg,g} c(x_{t,d,bg,g}) + \sum_{t,d,bg,g} tc(y_{t,d,bg,g}) + \sum_{t,d,bg,g} penalty(slack_{t,d}) \tag{15}$$

Subject to:

$$\sum_{g} x_{t,d,bg,g} = \sum_{g} x_{t-1,d,bg,g} - \sum_{g} losses_{t-1,d,bg,g} + \sum_{g} y_{t-2,d,bg,g} \qquad \forall\ t, d, bg \tag{16}$$

Constraint (16) bounds the primary flow for the entire system. This equation ensures that x is equal to the number of teachers from the previous year subtracting the number of losses and adding the number of TTC graduates.

$$\sum_{Z} x_{t,d,bg,g} = \sum_{Z} x_{t-1,d,bg,g} - \sum_{Z} losses_{t-1,d,bg,g} + \sum_{Z} y_{t-2,d,bg,g} \qquad \forall\ t, d, bg \tag{17}$$

Where Z is a subset of g either pr, se, hs. Constraint (17) allows the model to move teachers within education levels of primary, secondary, and upper secondary exclusively. This provides a more realistic hiring plan allowing provinces to shift teachers accordingly to fill gaps.

$$losses_{t,d,bg,g} \geq= \nu\% \times x_{t,d,bg,g} \tag{18}$$

Where ν is the expected proportion of losses. Constraint (18) allows the model to plan losses, or to fire while maintaining the minimum losses by grade as forecasted by data. In later versions of the model, the number of losses is run as a random sample for sensitivity analysis.

$$u = st_{2012} - Ordinality(t)\left(\frac{st_{2012} - st_{2025}}{number\ of\ time\ steps}\right) \tag{19}$$

$$x_{t,d,bg,g} \geq \frac{r_{t,d,bg,g}}{u} \tag{20}$$

Where *st* is the teacher student ratio. Equation (19) and Constraint (20) constrain each province by grade and gender to meet a function of student teacher ratio. The function is a linear model which begins in 2012 at the provinces current ratio and steps to the goal class size of 30 students per classroom by 2025.

$$\sum_{g,bg} y_{t,d,bg,g} + \sum_{g,bg} y_{t-1,d,bg,g} \leq k_d(1+.1)^{ordinality(t)} + slack_{t,d} \qquad \forall\ t,d \tag{21}$$

Constraint (21) is the capacity constraint for teacher training. Here a slack variable is used to account for a lack of capacity, providing elasticity and allowing model to solve. This slack variable accounts for the additional required capacity in the provinces training base. This capacity corresponds to the required number of training positions in TTC for the respective province beyond that of current or forecasted capacity. Capacity is assumed to grow at 10% compounded annually. This is a bit lower than the MOE goal for the next three years, but is an adequate fit for the out years.

Variables are positive

$$x_{t,d,bg,g} \geq 0 \tag{22}$$

$$y_{t,d,bg,g} \geq 0 \tag{23}$$