Simulating the Market for Protection

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Abstract

Konrad and Skaperdas (2012) develop an analytical model of the provision of security as a public good. The analysis of this "market for protection" begins with a population of "bandits" preying on "peasants". For clarity and tractability, the model makes a number of simplifying assumptions. An agent-based simulation has been developed to enable the systematic exploration of the implications of those assumptions. To achieve an initial level of confidence in the implementation of the analytical model, an analytical result defining an equilibrium condition is successfully compared with high-level results from the simulation.

The behavior of the simulation is explored in more detail to understand where it differs from the analytical model. The principal finding from this exploration provides details on the conditions under which the bandit and peasant populations will reach equilibrium. The analytical model predicts a single equilibrium where the populations of bandits and peasants are stable, independent of the number of peasants and bandits. The simulation finding is that this adjustment process is asymmetric. When bandits start out doing better than peasants, the population adjusts to an equilibrium as predicted by the analytical model. When peasants start out doing better than bandits, the analytically predicted equilibrium is not reached. This is interpreted as an effect of the movement from a continuous analysis to a discrete simulation, where the strategy of individual agents has an impact on the overall behavior of the population.

More generally, this work serves as an example of the combined use of both analytical models and agent-based simulations in exploring theoretical questions. The goal of the simulation is to aid understanding of the analytical model, by making more concrete the conditions under which the model does and does not hold. In turn, analytical techniques aid the development of the simulation by acting as a double-check on the simulation results, and by bringing focus to important combinations of parameters.

Analytical Model

The Market for Protection

Security is a pre-requisite for economic development and activity. Konrad and Skaperdas (2012) begin their analysis with the interaction of individual agents in anarchy, without any collective agreement or organization. *Peasants* spend their effort in productive activity, and consume some of their output to sustain themselves until the next period. *Bandits* produce no output, but prey on peasants, forcing the peasants to surrender their output for the bandits' consumption.

A peasant may spend some portion x of her unit effort on securing her output against bandits, spending the remainder of her effort (1 - x) on productive work. A *protection function*, $p(x)$, models this security effort; it converts security effort into effective protection of some proportion of a peasant's output, leaving the remainder to be surrendered to a bandit. Given continuous populations and a continuous, non-decreasing protection function, the analysis then identifies an optimal proportion of the peasant's overall effort that should be dedicated to security, and the conditions under which the population of bandits and peasants will reach an equilibrium, where all actors have the same average payoff.

The payoff to a peasant from the choice of protection proportion x is as follows:

$$U_p = p(x)(1 - x) \tag{1}$$

Given a total population N consisting of a number of bandits $N_b > 0$ and a number of peasants N_p, the payoff to a bandit from preying on a peasant is:

$$U_b = [1 - p(x)](1 - x)\frac{N_p}{N_b} \tag{2}$$

Given a peasant's optimal choice x^* of the protection proportion, with optimal utility U_p^*, an equilibrium is reached when the numbers of peasants and bandits adjust until bandits and peasants have the same payoff U_p^*=U_b^*. Anticipating the simulation implementation, we can think of agents shifting back and forth between the bandit and peasant roles until both roles have the same utility. The numbers of bandits and peasants are then given by:

$$\begin{aligned} N_p^* &= p(x^*)N \\ N_b^* &= [1 - p(x^*)]N \end{aligned} \tag{3}$$

Simulation

Model Assumptions and Simulation Overview

In simulating the market for protection (http://www.openabm.org/model/3851/version/2/view) we modify several of the assumptions of the analytical model. Populations are discrete rather than continuous. The protection function is given the specific functional form of a contest success function. Rather than assuming that all peasants use the same optimal protection proportion, peasants choose a protection proportion from the uniform distribution. In each period, agents are randomly matched for interaction, either by type (bandits only interact with peasants), or without regard for type (any agent may interact with any other).

Equilibrium is a concept from the continuous analysis, defined over a fixed total population, where the numbers of bandits and peasants reach a constant proportion. As implemented in this discrete simulation, equilibrium is defined as a state where the average payoffs to bandits and to peasants are equal, within a tolerance, and maintained for a configurable number of consecutive periods. A "role-shifting" dynamic is implemented, such that if the populations are not in equilibrium in a given period, a configurable percentage of the lower-performing role shifts to the better-performing role in the next period.

The simulation is written in Java, and its behavior is governed by a set of parameters. A general replication framework has also been written to enable the systematic exploration of the parameter space, and the replication of earlier results following changes to the simulation code. At a more granular level, automated unit tests help ensure the correct behavior of individual classes.

Simulation Details

The analytical model assumes that all peasants spend the same proportion x of their effort on protection. In our simulation, this is the first assumption that is relaxed; peasants are randomly allocated a protection proportion in increments of 0.05 over the interval [0,1].

The analytical model places some restrictions on the peasant's protection function, but does not give it a functional form. The simulation uses a contest function for the protection function, to model the interaction of players where the probability for a favorable outcome increases in the effort of the player. "A Contest Success Function (CSF) provides each player's probability of winning as a function of all players' efforts" (Skaperdas, 1996). In this case, the contest function is only dependent on the effort of the peasant. We introduce a parameter γ to model the defensive ability of the peasant. γ varies over the interval [0.5, 1.0], and the resulting protection increases as defensive ability increases.[1] [2]

$$p(x) = \frac{\gamma x}{\gamma x + (1-\gamma)(w)} \tag{4}$$

[1] In the simulation, $p(1)$ is forced to $= 1$, and $p(0) = 0$, to conform to its definition in the model.

[2] A weight w defaults to 1 in the current model, but will vary in later versions.

In each period, bandits and peasants are randomly assigned to interact. The *normal* interaction pattern is that bandits only interact with peasants. In the *any* interaction pattern, any agent may interact with any other. Moving from continuous to discrete populations, we assume that agents have a discrete number of interactions. In each period, one bandit may prey on at most one peasant, and one peasant may be preyed upon by at most one bandit.[3] This means a given peasant or bandit may or may not have an interaction in a given period; if the numbers of bandits and peasants are not the same, some agents will not have an interaction. If a bandit has no interaction, or interacts with another bandit, the payoff is 0. If a peasant has no interaction, or interacts with another peasant, the peasant retains all of its output, for a payoff of 1 - x.

The *role-shifting dynamic* implements the concept of changing the proportions of bandits and peasants in a fixed population, through imitation. A percentage of the population whose average payoff is lower at the end of each period, will shift to the opposite role for the next period. The worst-performing members of a given population role are those who shift roles. When roles shift, the newly-created agents may be given the appropriate parameters of the best-performing members of the new role, or the parameters may be randomly assigned. Once parameters are set for a given agent, they do not change over time.

The simulation does not currently include mutation of the population strategies. Stochasticity is introduced when populations are initially created, and during the interaction of agents. One of a set of seeds is used to generate all random numbers needed during execution of the simulation. This provides a deterministic sequence of pseudo-random numbers, which enables a given scenario to be repeated with exactly the same results for testing or replication.

Equilibrium is defined as a state maintained over a configurable number of consecutive periods (always 10, in these simulations), in which the average payoffs for the bandit and peasant populations are equal, within some configurable tolerance. In such a state, there is no shifting of agents from one population to the other, and the size of the overall population does not change.

The set of parameters of the simulation is the parameter space. This space has K dimensions, where K is the number of parameters. Parameters have different types, including integer, real, and boolean. A single parameter point can be thought of as a vector in the parameter space, and a single execution of the model with that parameter point is a scenario. A scenario set is the execution of all the scenarios corresponding to the points of a given parameter space.

A given scenario terminates under any of several conditions. If either the population of bandits or peasants drops to 0, it has become *extinct*, and there is no possibility of future interaction, so the scenario ends. If the number of bandits or peasants exceed a maximum

[3] The number of interactions is configurable; the simulation enables multiple bandits to prey upon multiple bandits, but no results from such configurations are discussed here.

level (configured to ensure that the simulation does not run out of available memory), this is taken as evidence that that population will grow indefinitely without limit, and the scenario ends.[4] If the populations of bandits and peasants are in equilibrium for a configurable number of consecutive periods, the system is interpreted as having reached equilibrium, and the scenario ends. Otherwise, the scenario ends after a configurable maximum number of periods has elapsed without any of the above conditions.

Analysis

Interaction of a Single Peasant and Bandit

The goal of the simulation is to replicate the analytical model where possible, and to generate behavior in areas where the analytical model is less tractable and does not provide results. The initial goal of the analysis is to verify it implements the expected behavior of the analytical model. This begins with analyzing the behavior of a single peasant, followed by the interaction of a single bandit and peasant.

In the analytical model, equilibrium is defined in the aggregate, but in a simulation, interactions are discrete. A given peasant either is or is not preyed upon in a given period. When preyed upon, the peasant keeps protected output $p(x)(1 - x)$ and surrenders unprotected output $[1 - p(x)](1 - x)$. Similarly, a bandit preying upon a peasant will receive the peasant's unprotected output $[1 - p(x)](1 - x)$, but if no peasant is available to prey upon, the bandit's payoff will be 0. Therefore, depending on the values of the protection proportion x and the number of peasants and bandits, some of both peasants and bandits may have much higher payoffs than others. For example, if a peasant has the minimum protection proportion, $x = 0$, and is not preyed upon, the payoff is 1 - 0 = 1, the maximum possible. If a bandit preys upon such a peasant, however, the bandit takes all the peasant's output, leaving 0. So a peasant with $x = 0$ may have either a maximum or a minimum payoff. In the aggregate, will such a population of peasants persist? We might expect that this difference in behavior will cause the simulation results to differ from the analytical predictions; we explore those differences next, in the analysis of the conditions under which an equilibrium is reached.

Optimal Protection Proportions and Equilibrium

Should we expect that as a group, peasants' protection proportion approaches the optimal value x^*? An equilibrium state is one where peasants choose the optimal value x^*, and the numbers of bandits and peasants have shifted such that both roles realize the same payoff. Where $p(x)$ is the contest function, the optimal value x^* is given by maximizing the payoff

[4]This condition is only possible with a dynamic where the total population grows. See Appendices 1 and 2 for a discussion of such a dynamic.

function:

$$U_p = \frac{\gamma x(1-x)}{\gamma x + (1-\gamma)} \tag{5}$$

For values of x in the interior of [0,1], and γ in its permissible range of [0.5,1], the denominator of this equation is always positive.[5] Maximizing this objective function gives a single root in the interval (0,1):

$$x^* = \frac{\gamma - 1 + \sqrt{1-\gamma}}{\gamma} \tag{6}$$

The second derivative test generates a negative value, confirming that this is a maximum.

Solving this equation for some representative values of γ and substituting into the various equations for a total population $N = 2000$, gives:

γ	x^*	$p(x^*)$	U_p^*	N_p^*	N_b^*
0.5	0.41	0.29	0.17	582	1418
0.75	0.33	0.5	0.33	995	1005
0.95	0.18	0.77	0.63	1548	452
1.0	0.0001	1.0	0.99	2000	0
1.0	0	0	0	0	2000

As gamma rises, the protection function becomes more and more effective; the peasant optimally spends less effort on protection, and utility rises correspondingly. At low protection levels, in equilibrium bandits substantially outnumber peasants; at high protection levels, peasants outnumber bandits.[6]

We would like to simulate this equilibrium behavior by looking at the impact of the role-shifting dynamic, which implements the concept of shifting the number of agents playing a given role, based on the average payoff of that role. In performing this validation, we varied several parameters:

- γ for the contest function $\in$ [0.5,1.0], varied in increments of 0.05
- Initial peasant population $\in$ {1000, 2000, 3000}

[5] We discussed the exterior solutions of 0 and 1 above. $p(1)$ fails immediately under the DST dynamic, discussed in Appendices 1 and 2. $p(0)$ is discussed further in the Findings on equilibrium.

[6] The last two rows address boundary conditions. As gamma approaches 1, the optimal protection proportion approaches 0, as the protection function approaches being perfectly effective. The value of x=0.0001 is not correct, but illustrates that as the protection proportion gets sufficiently small, the optimal population is all peasants, and their utility approaches 1. On the other hand, by specification from the analytical model, when x=0, the protection function is forced to be 0. This reverses the results; the optimal population is all bandits, but since they have no peasants to prey upon, their utility is 0.

- Initial bandit population $\in \{1000, 2000, 3000\}$
- Proportion of the lowest-performing role to shift to the better-performing role $\in \{0.5, 0.1, 0.2\}$
- New agents adopt the best-performing strategy of the role to which they are shifting

In combination with 10 seeds for pseudo-random numbers, this generated a space of 2970 parameter points. Equilibrium was defined by the maintenance of payoffs within a tolerance of .01 for 10 consecutive periods. Three regression analyses were done on the resulting data, confirming at a high level the analytical results above. The protection proportion was negatively associated with an increase in the γ for the contest function. That is, as the effectiveness of the security effort increases, the optimal protection proportion decreases. Secondly, at equilibrium, reached for 50% of the parameter points, the utility for agents was positively associated with an increase in γ. That is, all agents do better as peasant protection becomes more effective. Both these effects were highly significant, with P values of 10^{-4} or less. Because in the analytical model there is no dependence of the protection proportion on the numbers of peasants and bandits, a third regression analysis tested whether the optimal protection proportion was sensitive to initial conditions regarding the numbers of peasants and bandits. In one third of the cases, there were equal numbers of peasants and bandits; in one third of the cases, peasants outnumbered bandits; in the last third of the cases, bandits outnumbered peasants. This test failed to produce a significant effect, indicating that initial conditions have little impact.

In summary, at this high level of analysis, the simulation results confirm the predictions of the analytical model. In the Findings section, we will return to these results in more detail.

Findings

Asymmetric Distribution of Equilibria

The summary analysis from the comparison of optimal protection proportions and equilibrium confirmed that at a high level, the optimum protection proportion is negatively correlated (r = -.75) with the value of γ for the contest function. A detailed examination of these results, however, shows that this relationship is not simply linear. This is most easily seen with a new statistic, which shows that the *combination* of γ and the initial ratio of peasants to bandits is important in understanding the simulation's behavior. After each period of the simulation, the average payoffs for all bandits and all peasants is compared to determine in which direction roles will shift. If we look at the ratio of the average peasant payoffs to average bandit payoffs after the first period, we see an asymmetric distribution of equilibria around the payoff ratio value of 1.0 (Figure 1).[7]

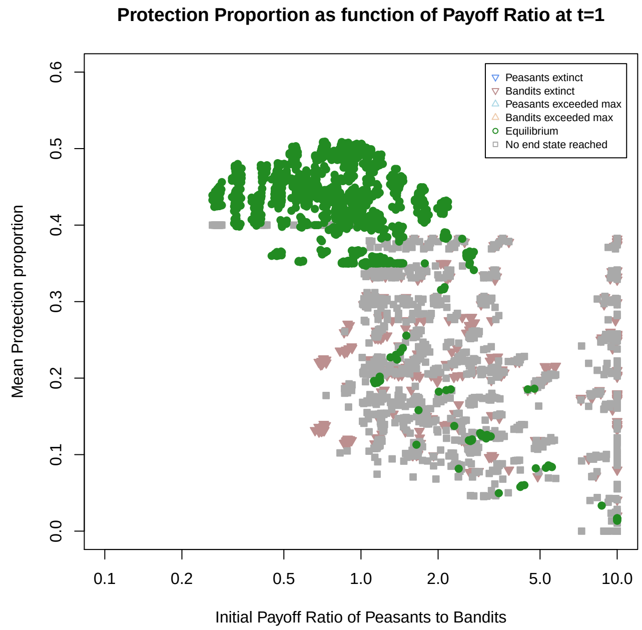

Figure 1: Protection Proportion as a function of Initial Ratio Peasant Payoffs to Bandit Payoffs

Initial Payoff Ratios Lead to Two Behavior Patterns

The asymmetric distribution of equilibria is due to two distinct behaviors in the simulation.

Low defensive ability: bandit payoffs are greater than peasant payoffs when γ is relatively low (peasant/bandit payoff ratio < 1). In Figure 1, this is the mass of points (42% of the total) to the left of 1.0, with protection proportions about 0.4 to 0.5. Most of these points show as green, indicating that equilibrium was reached.

Because their protection is minimally effective, peasants begin with lower payoffs than bandits, and begin shifting to the bandit role. As there are fewer peasants to prey upon in each period, payoffs drop for bandits; final utilities average 0.23.

Peasant protection proportions average 0.42, which is close to the analytical model optimum of 0.41; both low and high protection proportions lead to lower payoffs, and peasants with those proportions then shift to the bandit role. The initial ratio of peasants to bandits affects which way payoffs move for peasants. If peasants are equal to or outnumbered

[7]Note that the x-axis is a log scale. Ratios greater than 10 were forced to 10 (the highest ratio observed was 33); this affected about 6% of points.

by bandits, then their payoff rises modestly to the equilibrium level, because the peasants with lower payoffs shift to the bandit role, leaving the higher-payoff peasants, who raise the average. But when peasants outnumber bandits, their payoffs will drop over time, perhaps by as much as 50%. When peasants outnumber bandits, some are not preyed upon, so they keep all their output as a payoff, causing the average payoff to be higher. Because the payoff for bandits is even higher, however, some peasants shift to the bandit role in each period, which steadily lowers the average payoff of the remaining peasants. An example is shown in Figure 2. The number of bandits grows for the first 12 periods, but bandit utility drops. In the 13th period, payoffs for bandits and peasants are equal, within the equilibrium tolerance, and stay close to constant for the next 10 periods, which defines the equilibrium state.

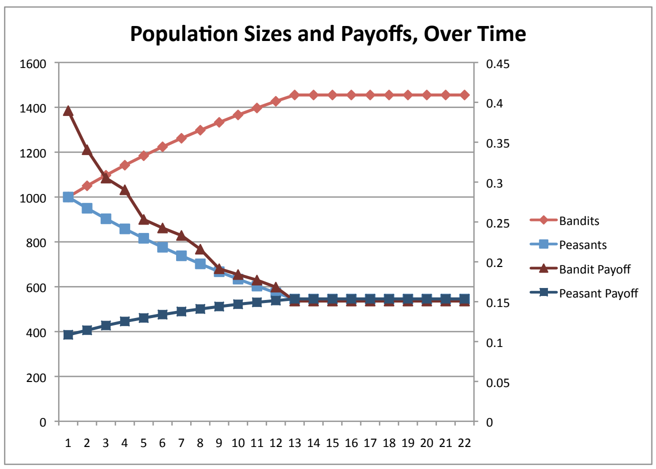

Figure 2: Equilibrium reached for initial Bandit Payoffs greater than Peasant Payoffs

Equilibrium is reached almost 90% of the time when initial bandit payoffs exceed initial peasant payoffs, and fairly quickly – on average, in 24 periods. Bandits outnumber peasants roughly 3 to 2 in the end state, regardless of the initial ratio of peasants to bandits.

High defensive ability: peasant payoffs are greater than bandit payoffs when γ is relatively high (peasant/bandit payoff ratio > 1). In Figure 1, this is the mass of points to the right of 1.0. Protection proportions are much more widely distributed for this mass (58% of the total). Many of the points show that no equilibrium was reached before the run limit of 100 periods was reached. In a smaller number of cases, equilibrium was reached or bandits went extinct.

Because their protection is very effective, peasants begin with higher payoffs than bandits, and bandits begin shifting to the peasant role. The new peasants are given the highest performing strategy of the existing peasant population. Over time, peasants generally come to outnumber bandits, at which point, the best-performing strategy becomes a protection proportion of x=0. Any peasant with this strategy who is not preyed-upon, will have the highest possible payoff (1), and subsequent new peasants will adopt this strategy, leading to more and more peasants with x=0. This has the effect of raising the average payoff for peasants, often so quickly that bandits cannot keep up. This "race to the top" frequently continues indefinitely, until the run limit expires, or all bandits convert to peasants and the bandit population is extinct. Utilities under this case are much higher than in the first case; bandits average 0.44 and peasants average 0.64. An example is shown in Figure 3. Payoffs start very close; in period 1, bandits do slightly better than peasants, but due to stochastic effects, bandit payoffs drop in period 2, and never recover. As bandits shift to peasants, bandit payoffs fluctuate, but they never do as well as peasants, and eventually the bandit population goes extinct.

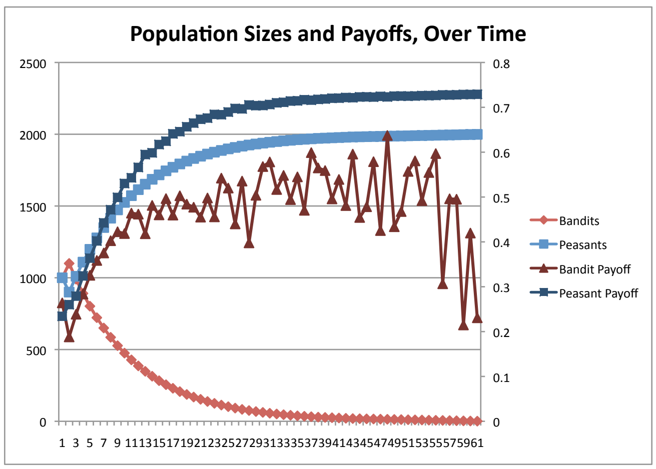

Figure 3: "Race to the top" ending in Bandit extinction, for initial Bandit Payoffs less than Peasant Payoffs

Intermediate values of γ, e.g., 0.75, lead to results between the high and low cases, but with more variability. Equilibrium is often reached, but sometimes bandits go extinct, particularly if peasants outnumber bandits at the outset.

Impact of Asymmetric Equilibria Distribution on Protection Proportions

The difference between these two classes of simulation outcome can be seen clearly in a histogram of the modes of the protection proportions chosen:

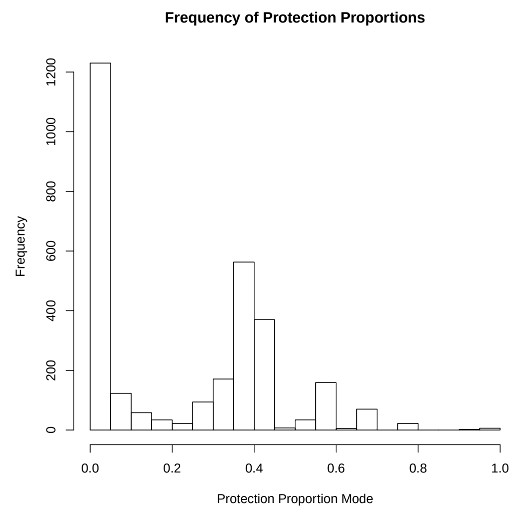

Figure 4: Distribution of the Mode values of Protection Proportion

This shows a clear bi-modal distribution. 0 and 0.35 are the two most common protection proportions. This bi-modal distribution of protection proportions in the simulation results is not predicted by the analytical model. The implications of this result are reviewed in the Discussion section.

These results reflect the assignment of the best-performing protection proportion to new peasants. The simulation was also run where assignment of the protection proportion to new peasants is random. This had no positive impact on the tendency of the simulation to reach equilibrium. What we observe then is that the most common protection proportion in these cases becomes random, reflecting the random assignment.[8]

Discussion

The asymmetric distribution of equilibria in the simulation results is an effect of the movement from a continuous analysis to a discrete simulation. In the analytical model, the analysis is done at the group level; the population of peasants is preyed upon by the population of bandits. All peasants are subject to the same level of banditry, and surrender their unprotected output uniformly. This puts uniform pressure on peasants, leading them to adopt the single optimum protection proportion.

[8]Appendix 2 documents the results when a second dynamic is combined with role-shifting. Although the asymmetry is not as pronounced, it is still present under these conditions, for the same reasons.

Asymmetrical Equilibrium Distribution under the Role-Shifting Dynamic

The discrete interaction of peasants and bandits in the simulation leads to the asymmetric equilibria distribution, not predicted by the analytical model. The difference stems from whether all peasants will be preyed upon or not. When all peasants are not preyed upon, a strategic component is introduced into the analysis. We can summarize the asymmetric behavior as if peasants had two strategies, "hope for the best" or "prepare for the worst". When bandits equal or outnumber peasants, peasants are better off playing it safe with the optimum strategy, clustered around 0.35-0.4. This leads to the best payoff when predation is close to inevitable ("prepare for the worst"). When peasants outnumber peasants, the aggregate best strategy is not to spend any effort on protection. Some peasants will not be preyed upon and will have the maximum payoff ("hope for the best"); others will be unfortunate enough to be preyed upon and will have 0 payoff. This situation is triggered when initial peasant payoffs are higher than bandit payoffs, which is in turn caused by more effective peasant defensive abilities (higher γ).

More specifically, when bandits outnumber peasants, a given peasant will encounter a bandit with probability 1, and will therefore realize a payoff of:

$$U_p = p(x)(1-x) \tag{7}$$

When peasants outnumber bandits, the expected payoff to a peasant is:

$$U_p = p(x)(1-x)\frac{N_b}{N_p} + (1-x)(1-\frac{N_b}{N_p}) \tag{8}$$

When the average payoff to peasants is higher than the average payoff to bandits, i.e., γ is sufficiently high, eventually sufficient bandits will shift to the peasant role so that the second payoff calculation applies. If x=0, this second payoff reduces to:

$$U_p = 1 - \frac{N_b}{N_p} \tag{9}$$

Eventually, there will be sufficiently more peasants than bandits so that the payoff in (9) is greater than the payoff in (7), so new peasants are better off choosing x=0. This generates a free rider effect; the original set of peasants does best with x^*, paying a constant cost for protection, but later peasants can take advantage of the fact that the existing mass of peasants outperforms and therefore outnumbers bandits, and choose a protection proportion that minimizes their costs and maximizes their retained output.[9][10]

[9] Peasants don't actually choose protection proportions; the proportions are assigned, but the impact is the same.

[10] This is most clearly seen when new peasants get the best-performing strategy x=0, but the mechanism is the same when the new peasant gets a random protection proportion. In that case, the final most frequent protection proportion can't be predicted.

Another way to think about this is that the role-shifting dynamic is the mechanism that drives the peasant population toward the optimal protection proportion x^*, by removing poor-performing protection proportions from the population. This can only work when peasants are moving to the bandit role, however. When bandits are moving to the peasant role there is no selection pressure on the protection proportion of new peasants, so there will be no convergence to an optimal x^*.

Conclusion

Simulation of the analytical model of the market for protection allows us to see the significant impact of moving from an assumption of continuous populations to discrete populations. Instead of a single equilibrium, we see a distribution of equilibria that has at least two modes. As the discrete assumption is more realistic, this suggests that a single equilibrium will not be seen under real-world conditions.

This study shows the usefulness of working with both analytical models and agent-based simulations. Analytical models provide simple, clear sets of predictions, which can first be used to give some assurance of the high-level correctness of the simulation. Then, the details of the simulation can be used to explore the implications of the assumptions built into the analytical model.

Appendices

Appendix 1: Evolutionary Game Theory Analysis

The MFP analytical model defines an equilibrium for a static total population. In this appendix and Appendix 2 we add a second dynamic to the simulation to enable the numbers of agents to rise and fall. An evolutionary game theory analysis is then performed, and compared with the simulation results to verify that the simulation behaves as expected as the population state changes.

In addition to the role-shifting dynamic, we define a *die/survive/thrive dynamic* (DST) for the evolution of populations when the total population is not fixed. All agents live for a single period. An agent dies without descendants if its payoff falls below the *survive* threshold $s \in [0,1]$; at or above this threshold a single descendant is created. If an agent's payoff exceeds a higher, *thrive* threshold $s \in [0,1]$, $t \geq s$, then it has two descendants. Descendants inherit the characteristics of their parent. Peasants inherit a protection proportion, and a protection function. For the simulations analyzed here, bandits have only a single strategy (always prey upon a single available peasant); there is no variation between generations. Payoffs are not inherited; each agent begins the period with a payoff of 0.

Evolutionary game theory gives us additional tools to analyze the evolution of a set of populations from a range of initial conditions. If this matches the behavior of the simulation reasonably closely, we will gain additional confidence in the simulation. In particular, in this analysis, we will look at the impact of the DST dynamic on the behavior of the simulation. To simplify the analysis, we hold the role-shifting behavior constant, by disabling the role-shifting dynamic. Also, although evolutionary game theory can be applied to either constant or changing overall populations, in this analysis the overall population is held fixed.

Evolutionary game theory allows us to analyze the evolution of a population over time. The individual players play pure strategies, and we analyze the change over time in the proportions of the population playing each pure strategy. To keep the analysis reasonably tractable, we define three strategies: "B" is a bandit strategy, "H" is a peasant strategy where the peasant picks a high protection proportion, and "L" is a peasant strategy where the peasant picks a low protection proportion. To simplify the analysis, we use the "any" interaction pattern, so we have a single population, where any combination of interaction among these three strategies is possible.

The DST dynamic is a two-step process. First we calculate the payoff to each agent from their interaction, and then we calculate the number of descendants that result. Evolutionary game theory is only interested in the second step – the change in the number of players playing a given strategy from one generation in the next. We pick parameters for convenience, as follows:

- High protection proportion: 0.7

- Low protection proportion: 0.1
- γ for contest function: 0.5

This results in the following payoff matrix, expressed as utilities (as the game is symmetric, payoffs are only listed for the row player):

	B	L	H
B	0	0.81	0.17
L	0.08	0.9	0.9
H	0.12	0.3	0.3

We then set the survive and thrive thresholds as follows:

- Survive threshold: 0.1
- Thrive threshold: 0.4

This generates a new game, where payoffs are interpreted as the number of descendants:

	B	L	H
B	0	2	1
L	0	2	2
H	1	1	1

Peasants with High protection always have one descendant. They can always protect enough of their output to survive bandit predation, but they spend so much effort doing so, that they never thrive; even when interacting with another peasant, so much effort is spent on protection that their entire output does not exceed the thrive threshold. Peasants with Low protection, on the other hand, are in a "feast or famine" situation: if they interact with another peasant, they will keep all their output and thrive with two descendants, but when preyed upon, they have too little protection to survive and will die without descendants. Bandits have a mixed bag of outcomes: when they prey on a Low peasant they will thrive with two descendants; when they prey on a High peasant they will only survive, with one descendant; when they encounter another Bandit, neither has any output, so both die with zero descendants.

We define a mass of players N where the numbers of players playing the three strategies B, H and L are m_b, m_h, and m_l. Then the proportion of the population playing a given strategy i is:

$$x_i = \frac{m_i}{N} \tag{10}$$

The fitness function $F_i(x)$ for a given strategy i gives the expected payoff to that strategy given a current population state (set of population proportions) x:

$$F_i(x) = x_b u(i, B) + x_h u(i, H) + x_l u(i, L) \tag{11}$$

Then the proportion of the population playing a given strategy at time $t + \delta$ is the following function of the proportions at time t:

$$\begin{aligned} x_i(t+\delta) &= \frac{m_i(t) \cdot F_i(x)}{m_b(t) \cdot F_b(x) + m_l(t) \cdot F_l(x) + m_h(t) \cdot F_h(x)} \cdot \frac{\frac{1}{N}}{\frac{1}{N}} \\ &= \frac{x_i(t) \cdot F_i(x)}{x_b(t) \cdot F_b(x) + x_l(t) \cdot F_l(x) + x_h(t) \cdot F_h(x)} \\ &= \frac{x_i(t) \cdot F_i(x)}{\bar{F}(x)} \\ &\approx \frac{x_i(t) \cdot (F_i(x) + \alpha)}{\bar{F}(x) + \alpha} \end{aligned} \tag{12}$$

This is the discrete-time replicator dynamic, where α is a constant added to all entries in the payoff matrix to ensure that all payoffs are positive (and thus the dynamic is defined on all areas of the simplex), a common and necessary move for those utilizing this dynamic. This α is interpreted as "background fitness", the amount of descendants an individual would be expected to have before the effects of the strategic interaction in question are taken into account. The payoffs from the game are then the number of additional offspring gained from the interaction. We note that the discrete time replicator dynamic is not invariant to such changes, but as the purpose of the game-theoretic analysis here is to check the general accuracy of the simulation and focus attention on particular parameters of interest, the important fact is that the locations of the equilibria are unchanged (although the stability of the equilibria may change)(Rowe et al., 1985).

This assumes distinct time-steps and that the population reproduces all at one time. If we let the overlap between generations approach infinity, this approaches the continuous-time replicator dynamic (Weibull, 1997, p. 124-6)[11], whose behavior is well-understood. The continuous time replicator dynamic is invariant to the addition of a constant to all payoffs, so if we wish we can ignore the background fitness and again use the original interpretation of the "die-survive-thrive" thresholds. Graphing the replicator dynamics in Mathematica generates the following simplex diagram.

[11] Note that a background birth and death rate is assumed.

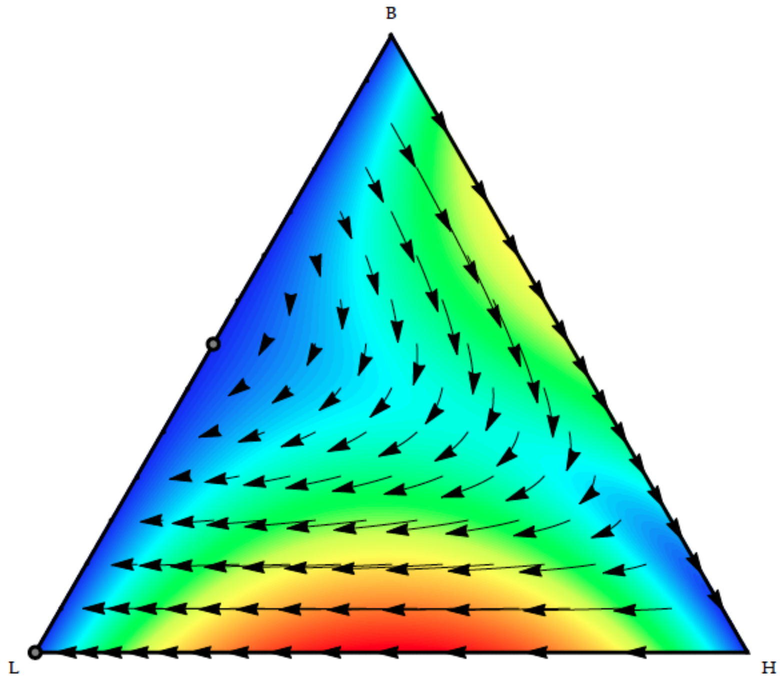

Figure 5: Nash Equilibria

Figure 5 shows the movement of various population states from various initial conditions, and shows two Nash equilibria (NE) for x_b, x_l, x_h where no population can do better by a unilateral deviation in strategy: (0,1,0) and ($\frac{1}{2}$,$\frac{1}{2}$,0). In the evolutionary context, however, we ask whether a Nash equilibrium is also an *evolutionary stable state* (ESS). An ESS is a population state x around which there exists a local neighborhood of states where the average payoff to the state x is higher than the average payoff to any state y whose proportions are some small mutation ϵ of x. This captures the notion that small perturbations in the proportion of strategies, perhaps due to random mutation, will not result in movement away from the ESS. Any ESS is a NE, but not all NE are ESS. To determine if (0,1,0) is an ESS, we suppose a state y such that y is an alternative best response (BR) to x. Then we have:

$$\begin{aligned} u((0,1,0),(0,1,0)) &= u((\epsilon,1-\epsilon,0),(0,1,0)) \\ 2 &= 2\epsilon + 2(1-\epsilon) \\ 2 &= 2 \end{aligned} \tag{13}$$

And x is not a better reply to y than y is to itself:

$$\begin{aligned} u((0,1,0),(\epsilon,1-\epsilon,0)) &= u((\epsilon,1-\epsilon,0),(\epsilon,1-\epsilon,0)) \\ 0\epsilon+2(1-\epsilon) &= 0\epsilon^2+2\epsilon(1-\epsilon)+0\epsilon(1-\epsilon)+2(1-\epsilon)^2 \\ 2(1-\epsilon) &= 2(\epsilon-\epsilon^2)+2(1-2\epsilon+\epsilon^2) \\ 2(1-\epsilon) &= 2(1-\epsilon) \end{aligned} \tag{14}$$

Therefore, y could successfully invade x, and (0,1,0) is not an ESS. Through a similar analysis, ($\frac{1}{2}$,$\frac{1}{2}$,0) is also not an ESS. Looking at the simplex diagram, we can interpret these findings as follows. The lower left corner corresponds to a population of just Low peasants. Interacting only with themselves, these peasants always thrive, so they receive the maximum possible payoff. On the other hand, this population can be invaded by Bandits. Paradoxically, it turns out that in a state where there are no High peasants, both Bandits and Low peasants have the same fitness function:

$$\begin{aligned} F_b(x) &= x_b u(B,B)+x_h u(B,H)+x_l u(B,L) \\ &= x_b(0)+(0)(1)+x_l(2) \\ &= 2x_l \end{aligned} \tag{15}$$

$$\begin{aligned} F_l(x) &= x_b u(L,B)+x_h u(L,H)+x_l u(L,L) \\ &= x_b(0)+(0)(1)+x_l(2) \\ &= 2x_l \end{aligned} \tag{16}$$

This means that any invasion of Bandits into a pure population of Low peasants will, on average, persist indefinitely. When either a Bandit or a Low peasant encounters a peasant, the first agent receives 2, but when either a Bandit or a Low peasant encounters a bandit, the first agent receives 0. More generally, by the same logic, any point on the entire left edge of the simplex triangle is a rest point (see Figure 6), one where the dynamics of the system do not drive the population away from its current state. But none of these represents an ESS, because any change in the proportions of Bandits and Low peasants is also a rest point.

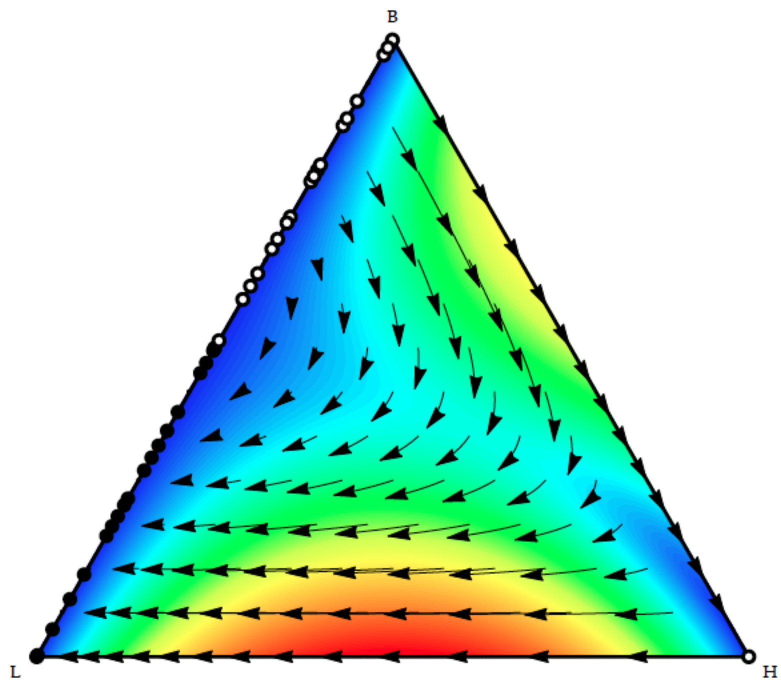

Figure 6: Rest Points

How do we interpret the rest of the simplex? The right edge of the triangle represents mixtures of Bandits and High peasants. Any such mixtures will move towards a population of just High peasants, because High peasants have one descendant no matter who they interact with, but Bandits only have one descendant when they interact with High peasants, and zero descendants if they interact with other Bandits.

In the interior of the simplex, the dynamic near the top of the simplex mirrors that on the right-hand edge. In these states there are a lot of Bandits, who will frequently interact with each other, leaving zero descendants. Consequently, the proportion of High peasants will rise rapidly. Note that the absolute number of High peasants never changes – they always just survive – but their *relative* numbers (their proportion of the entire population) rises, due to the number of Bandits who die.

Interior states imply that there are at least some Low peasants as well, and this eventually changes the trajectory of the dynamic. Starting in states near the top, with many Bandits, the Low peasants don't fare well, as their odds of encountering a Bandit are high, and they have zero descendants. However, as the proportion of Bandits in the overall population drops, the Low peasants start to have an advantage over both Bandits and High peasants. For example, in the state $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, where there are equal numbers of all three strategies, in any three interactions, Low peasants are likely to encounter two peasants, resulting in 4 descendants. On average Bandits will encounter another Bandit, a Low peasant, and a High peasant, for 3 descendants. No matter who the High peasants encounter, they will have 3 descendants. This general pattern favoring the Low peasants in the interior means that eventually, the dynamic will move toward the left side (the Low vertex), as the proportion of Low peasants steadily grows. This also explains the difference between the open rest points on the upper left edge, and the closed rest points on the lower left edge. In the upper areas, any mutation that introduces High peasants will lead to some initial movement toward the High vertex, as Bandits do worse relative to High peasants. But once the proportion of Low peasants is greater than that of Bandits, in the lower areas, introduction of High peasants has no lasting impact; the Low peasants have more descendants than either High peasants or Bandits, and the dynamic drives back to the rest point.

Simulation Results Compared to the Game Theory Analysis

The simulation was run 17 times with an N of 6400, varying the proportions of Bandits, Low peasants and High peasants in increments of 400, resulting in 153 parameter points, which were then graphed using R, resulting in Figure 7.

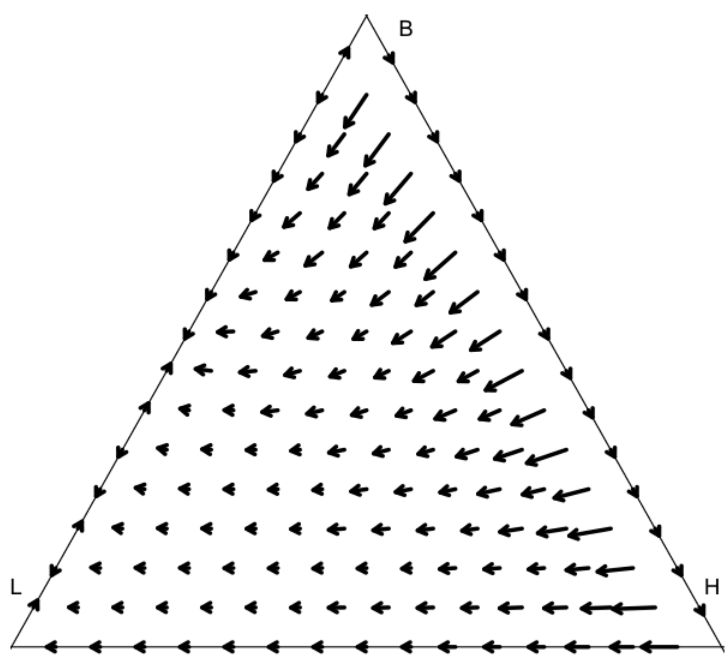

Figure 7: Simulation Dynamics

Due to limitations in the current logging interface of the simulation, this analysis results in a different depiction of the dynamic than the Mathematica output. For each point, the final population proportion was compared with the initial population proportion, and the direction of movement derived from this difference. Ideally, the comparison should be done between periods $t+1$ and t, rather than between the last and first periods. The underlying data are consistent with the Mathematica output, however, as can be seen by looking at the details from the simulation of a representative single parameter point, in Figure 8, below.

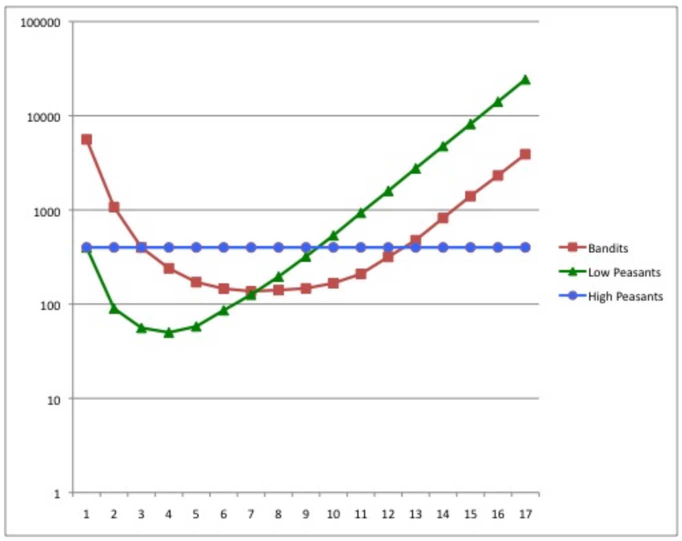

Figure 8: 5600 Bandits, 400 High, 400 Low, 17 periods (log scale)

This parameter point is near the top of the simplex. Between time $t = 1$ and $t = 2$, the greatest movement in population proportions is from Bandit toward High peasant, as many Bandits (roughly 4500 – note that the graph has a log scale) encounter other Bandits and leave zero descendants. This corresponds to the large movement seen at the top of the Mathematica output toward the High vertex. Subsequent periods are equivalent to points lower and to the right of the Mathematica simplex, where the High peasants are the predominant proportion of the population by period $t = 4$. However, by period $t = 7$, the number of Bandits has dropped sufficiently that the advantage of the Low peasants becomes felt. After that, the Low peasants grow rapidly, corresponding to the set of leftward-pointing horizontal arrows in the Mathematical simplex. The arrow in the simulation graph captures the difference in proportions from time $t = 1$ to time $t = 17$ – a net strong movement from a preponderance of Bandits to an even greater preponderance of Low peasants.

Appendix 2: Equilibrium under DST and Role-Shifting Dynamics

Having analyzed separately the behavior of the DST dynamic and the role-shifting dynamic, the two were combined in scenarios defined by the following parameter space:

- γ for the contest function $\in$ [0.5,1.0], varied in increments of 0.25
- Initial peasant population: 1000
- Initial bandit population: 1000
- Proportion of the lowest-performing role to shift to the better-performing role: .05
- Survive threshold $\in \{0.1, 0.15, 0.2, 0.24\}$
- Thrive threshold $\in \{0.25, 0.3, 0.35, 0.4, 0.45, 0.5\}$
- New agents adopt the best-performing strategy of the role to which they are shifting

In each period, the role-shifting dynamic was invoked first, followed by the DST dynamic.[12] Random seeds and equilibrium definitions were the same as in previous scenario sets. This generated a space of 5040 parameter points.

The regression analyses previously performed on the role-shifting scenario set were done on these data, with results similar to those from role-shifting alone.[13] The protection proportion was negatively associated with an increase in the γ for the contest function (Figure 9). The utility for agents was positively associated with an increase in γ.

[12]The dynamics will produce different results run in the opposite order. This hasn't been analyzed, but should lead to similar conclusions.

[13]The analysis of the ratio of initial numbers of peasants to bandits was not done, as neither initial population varied; both were set at 1000.

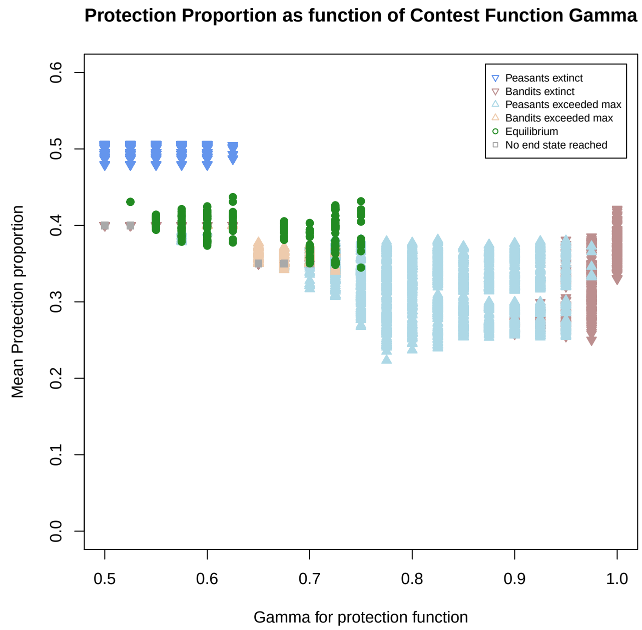

Figure 9: Protection Proportion as a function of Contest Function Gamma, Both Dynamics

As a general observation, the population states evolved much more quickly under the effects of both dynamics than under the effect of the role-shifting dynamic alone. Scenario end states were reached twice as quickly; on average, end states were reached in 27 periods for the combined dynamics, as opposed to 54 periods for role-shifting alone. Equilibrium was reached much less often; an equilibrium end state reached for 7% of the parameter points, as opposed to 50% for role-shifting alone. The percent of equilibria reached when the initial ratio of peasant to bandit payoffs was less than 1.0 was 100% under the combined equilibria, as opposed to 75% under role-shifting alone. This difference is likely to be due to the more rapid evolution toward end states under the combined dynamics.

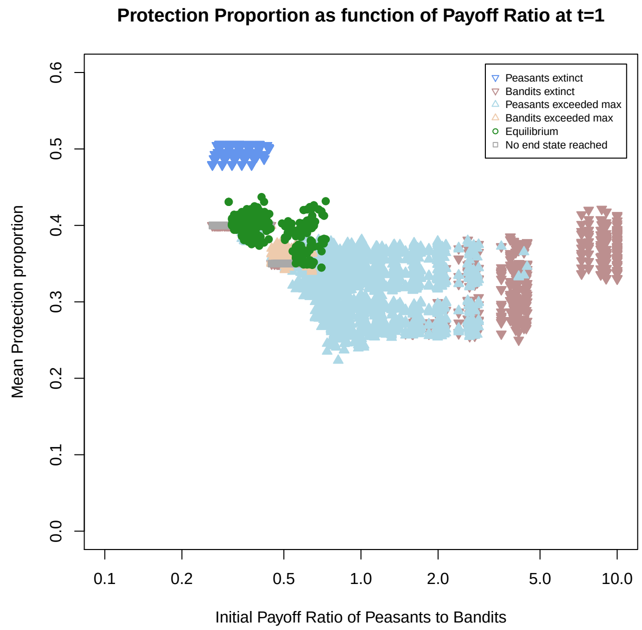

Figure 10: Protection Proportion as a function of Initial Ratio Peasant Payoffs to Bandit Payoffs, Both Dynamics

Figure 10 shows protection proportions and the distribution of end states under both dynamics (log scale). There is a pattern to the distribution of end states, as follows:

- Peasants go extinct in the first period for 10% of parameter points, when the survive threshold is relatively high, $s = .25$, and their defensive ability γ is low. Because the peasant population has no time to evolve, the initial random distribution of protection proportions generates a mean protection proportion around 0.5.

- Bandits go extinct for 20% of parameter points. This happens very quickly (less than 10 periods) when γ is high, similar to what is seen under role-shifting alone. Bandits also sometimes go extinct over many (more than 80) periods when γ is low but the thrive threshold is high, $t > 0.35$. Neither population thrives. Bandits do slightly better, but the numbers in both populations drop gradually. Stochastic effects may decide which population goes extinct first.

- Peasants grow to maximum for 45% of parameter points, when γ is in a broad middle range, from 0.575 to 0.825, and the thrive threshold is not low, $t > 0.3$. The ratio of initial payoffs for peasants to bandits has less impact on the outcome; the division of ratios greater than or less than 1.0 is almost even. The number of periods to reach maximum ranges from 6 to 99 periods, with a mean of 9 periods.

- Bandits grow to maximum for 9% of parameter points, when γ is in a middle range, from 0.6 to 0.725, and the thrive threshold is low, $t < 0.4$. The ratio of initial payoffs for peasants to bandits is always less than 1.0. Both populations grow steadily, but bandits do consistently better and reach maximum first.

- Equilibrium is reached for 7% of parameter points, when γ is in a low to middle range, from 0.525 to 0.75. The ratio of initial payoffs for peasants to bandits is always less than 1.0. Most of the time, both populations are lower than their initial levels; bandits average 620 and peasants average 435. Frequently, the number of bandits is twice the number of peasants. In this pattern, peasants consistently survive, while bandits thrive, producing two descendants. One descendant encounters a peasant and thrives; the other fails to find a peasant to prey upon, and dies.

- No end state is reached by the run limit of 100 periods for 9% of the parameter points. γ is less than 0.725, and the ratio of initial payoffs for peasants to bandits is always less than 1.0. These cases resemble the equilibrium cases, but often both populations continue to drop slowly to very low levels; if the run limit were not reached, one or the other population would go extinct.

Figure 11 shows the distribution of the mode of protection proportions. A similar asymmetry of equilibrium distributions is present as for role-shifting alone, but slightly less pronounced. The distribution is still bi-modal, but when two dynamics are present, the optimal x^* around 0.4 is the more common value. Less common, but still very frequent is $x = 0.05$. Note that $x = 0$ is uncommon, unlike under role-shifting alone.

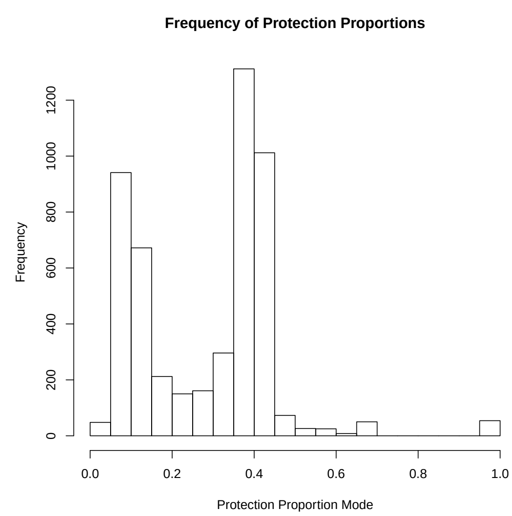

Figure 11: Distribution of the Mode values of Protection Proportion, Both Dynamics

Looking at individual scenarios, when an end state is reached quickly, there tends to be a broad range of protection proportions. High proportions are absent or less numerous, as they will perform worst; 0 will also be less numerous, when bandits outnumber peasants. When the end state is reached after many periods, and the population of peasants rises and falls multiple times, the distribution of protection proportions ends in a narrow range around the optimal x^*. Figure 12 illustrates this for a partial scenario (the final few periods are not graphed for clarity). At the conclusion of this scenario, 85% of peasants have a protection proportion of 0.35; the only other protection proportion present is 0.3, for the remaining 15% of peasants.

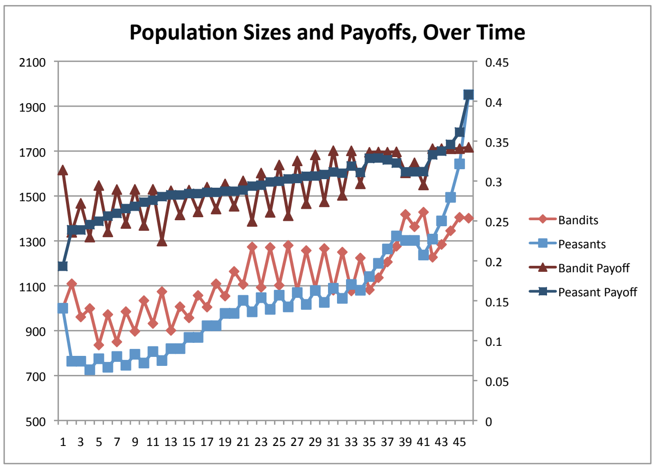

Figure 12: Fluctuation in Peasant population over many periods

Asymmetrical Equilibrium Distribution under both Role-Shifting and DST Dynamics

The addition of the DST dynamic does not alter the fundamental mechanism that leads to asymmetrical equilibria, but does modify its impact. A comparison of Figures 11 and 4 shows that the optimal protection proportion for peasants is reached more often under the combination of both dynamics than under role-shifting alone. A principal reason for this is that peasants die out if their payoffs do not reach the survive threshold. Peasants with protection proportions of 1 (or 0, if preyed upon) will always die out. This begins to narrow the range of protection proportions. If an end state is reached quickly, however, the range may of protection proportions may stay quite broad.

In a subset of cases, the interaction of the two dynamics may lead to a fluctuation in the peasant population, as shown in Figure 12. This fluctuation in turn is due to the relative fluctuation in bandit and peasant payoffs. Although peasant payoffs rise fairly steadily, bandit payoffs fluctuate regularly, higher and then lower than peasant payoffs.

Whenever bandit payoffs fall below peasant payoffs, peasants will move via role-shifting to the bandit role, eliminating less-efficient protection proportions. When the direction of movement reverses, and bandits move to the peasant role, they adopt the best-performing strategy. Since bandits outnumber peasants in almost every period, as we have seen this proportion for the new peasants will be near the optimal x^*. The combination of these two processes has a very pronounced focusing effect – the range of protection proportions becomes steadily narrower, around the optimal x^*.[14]

The combination of the DST dynamic, and cases where populations and payoffs repeatedly fluctuate, accounts for the greater proportion of parameter points under the combined dynamics where the protection proportion approaches the optimal value, than under role-shifting alone.

[14]Eventually in this scenario and others like it, peasant protection becomes so optimal that the peasant population and payoffs grow rapidly past the bandits, to maximum. The last few periods were not depicted to avoid swamping the detail of the earlier periods.

Appendix 3: Market for Protection ODD

Overview

Purpose

Konrad and Skaperdas (2012) develop an analytical model of the provision of security as a public good. The analysis of this "market for protection" begins with a population of "bandits" preying on "peasants" interacting in "anarchy". An agent-based simulation (MFPsim) has been developed to better understand the analytical model (MFP). In particular, if the analytical model is re-implemented as an agent-based simulation, do the conclusions of the analytical model still hold? For example, what is the effect of changing from continuous populations to discrete populations?

A peasant may spend some portion x of her unit effort on securing her output against bandits, spending the remainder of her effort (1 - x) on productive work. A protection function, $p(x)$, models this security effort; it converts security effort into effective protection of some proportion of a peasant's output, leaving the remainder to be surrendered to a bandit. Given continuous populations and a continuous, non-decreasing protection function, the MFP analysis then identifies an optimal proportion of the peasant's overall effort that should be dedicated to security, and the conditions under which the population of bandits and peasants will reach an equilibrium, where all actors have the same average payoff.

MFPsim modifies the MFP model as follows:

- Populations of bandits and peasants are discrete rather than continuous.
- The protection function is given the specific functional form of a contest success function.
- Peasants are randomly assigned a protection proportion.

In MFPsim, equilibrium is defined as a state where the average payoffs to bandits and to peasants are equal, within a tolerance, and maintained for a configurable number of consecutive periods. When the simulation reaches an end-state, the distribution of protection proportions in the peasant population can be compared to the optimal protection proportion predicted by the MFP model.

A technical purpose of the simulation is to enable future extensions to the simulation in a straight-forward manner to accommodate the remainder of the analytical model, where the original condition of anarchy is modified to introduce various forms of collective organization.

State Variables and Scales

The behavior of individual Peasants varies according to their protection proportions:

- *protectionProportion*: the amount of the peasant's unit effort devoted to defending its output, $\in$ [0,1]. Once set, the protection proportion stays constant for the lifetime of the peasant. Note: although the domain is continuous, during creation of the peasant population, the Protection Model allocates peasant proportions in bins, at particular values within the domain, to simplify the subsequent analysis.

All other state in MFPsim consists of a set of global parameters, whose values are set by the Protection Model. These are discussed in the Submodels section.

Process Overview and scheduling

- When a peasant and a bandit interact, the peasant's payoff is:

$$U_p = p(x)(1 - x) \tag{17}$$

  The payoff to a bandit is:

$$U_b = [1 - p(x)](1 - x) \tag{18}$$

  That means that the peasant keeps a proportion $p(x)$ of the productive output 1 - x, and the remainder of that productive output is surrendered to the bandit. If a peasant does not have an interaction with a peasant in a given period, the peasant retains all output 1 - x; if a bandit does not have an interaction in a given period, the bandit's payoff is 0.

- The protection function $p(x)$ is given the functional form of a contest success function:

$$p(x) = \frac{\gamma x}{\gamma x + (1 - \gamma)} \tag{19}$$

  The parameter γ (CONTEST_FUNCTION_GAMMA) is interpreted as the "defensive ability" of the peasant.

- In each period, the population of bandits and peasants are randomly matched 1:1 for interaction. Payoffs for each agent are calculated, and one or more Dynamics are invoked, resulting in a new population of bandits and peasants. A new period commences, with payoffs starting at 0; payoffs do not accumulate. This process continues until one of the following end states occurs:
  - Peasants go extinct: peasant population drops to 0.
  - Bandits go extinct: bandit population drops to 0.
  - Peasants go to maximum: peasant population exceeds MAXIMUM_POPULATION_SIZE
  - Bandits go to maximum: bandit population exceeds MAXIMUM_POPULATION_SIZE

- Equilibrium: peasants and bandits have the same average payoff within a tolerance PAYOFF_DISCREPANCY_TOLERANCE, for consecutive periods EQUILIBRIUM_NUMBER_PERIODS_WITHOUT_ADJUSTMENT.
- Run limit exceeded: the number of periods without reaching one of the above end states reaches RUN_LIMIT.

When an end state is reached, the scenario ends.

- There are two dynamics that may be applied to a population, resulting in a new population for the next period.[15]
  - Die/Survive/Thrive (DST) Dynamic: this dynamic may result in the total population of agents changing in size, and is defined by two thresholds. Agents whose payoff is not equal to or greater than the "survive" threshold (SURVIVE_THRESHOLD) die without descendants. An agent whose payoff is equal to or exceeds the survive threshold has one descendant, unless the payoff is equal to or exceeds the "thrive" threshold (THRIVE_THRESHOLD), when the agent has two descendants. A descendant inherits the strategy of its parent; specifically, a peasant in the new population inherits the protection proportion of its parent.[16]
  - Role-Shifting Dynamic: this dynamic results in a percentage of the lowest-performing role shifting to the better-performing role. The total population of agents does not change. Role performance is defined by the average payoff for all members of a role in each period. If the difference between the average payoffs is greater than PAYOFF_DISCREPANCY_TOLERANCE, then a percentage ADJUSTMENT_FACTOR_PERCENTAGE of the lower-performing agents shift to the other role. When bandits shift to the peasant role, they are given a protection proportion determined by flag NEW_PEASANT_GETS_BEST_PROTECTION_PROPORTION. If true, they get the current period's best performing protection proportion; otherwise, they get a randomly allocated protection proportion. Fractional adjustments less than 1 are rounded to 1, but fractions for numbers above 1 are rounded down to the next integer.

[15] Current implementation: DST dynamic is always invoked; Role-shifting dynamic may then optionally be invoked. The order is not reversible.

[16] Current implementation: The DST dynamic is always invoked, but may be effectively disabled by setting the survive threshold to 0 and the thrive threshold to 1. There is a known defect with this implementation, where peasants with $x = 0$ will thrive if not preyed upon.

Design Concepts

Emergence

The equilibria of the bandit / peasant population, and the most prevalent protection proportions, emerge from the individual interactions of bandits and peasants.

Adaptation

Agents do not adapt their behavior, nor do their strategies mutate. Once assigned, a peasant and its descendants keep the same protection proportion. Bandits do not have individual strategies.

Fitness/Objectives

Agents attempt to maximize their payoffs (utilities) in their interactions with other agents. But this behavior is determined by the combination of assigned protection proportion, and the matching pattern for interaction; there is no latitude for individual decision at the moment of interaction.

Prediction

Agents do not predict the consequences of their actions.

Sensing

Agents do not sense their environment. They are not placed on a spacial grid; their interaction patterns are determined randomly and externally.

Interactions

There are two interaction patterns, determined by flag NORMAL_INTERACTION_PATTERN. If true, bandits are always paired with peasants. If false, any agent may be paired with any other agent. The latter setting is used to create a single population for the purposes of evolutionary game theory analysis.

The populations are matched randomly in each period, with one agent interacting with one other agent, until one population is exhausted. The remaining agents have no interaction in that period, and receive a payoff as defined in the Process Overview.

Stochasticity

A single long integer is used as a seed for a pseudo-random number generator. This is used first to allocate protection proportions randomly when the population of peasants is first built at the beginning of the scenario. In each period, the random number generator is then used to shuffle one population, whose members are then selected one at a time to interact with the next member of the other population. When bandits move to the peasant role under the Role-Shifting Dynamic, and if NEW_PEASANT_GETS_BEST_PROTECTION_PROPORTION is false, the random number generator is used to assign a protection proportion to the new peasant.

Collectives

There is no collective or social organization to either the bandit or peasant populations; agents interact randomly, and are not affected by the interactions of others.

Observation

Summary statistics are gathered for every scenario, as one record in a file formatted as comma-separated values. Each record contains the values of each parameter, the parameter point, which defines a single scenario. The file thus records the results of traversing the parameter space, as a collection of scenarios, termed a "scenario set". In addition to the parameter values, each record includes these summary statistics:

- Scenario number: unique integer for each scenario in the scenario set
- Stop reason code: integer code defining for which of the six reasons the scenario stopped execution.
- Period: period in which the scenario stopped.
- Numbers of bandits and peasants, before and after replication in the final period of the scenario.
- Average bandit and peasant payoffs
- Discrepancy between average bandit and peasant payoffs
- Adjustment, if any, between the bandit and peasant roles. Positive integers: bandits to peasants; negative integers: peasants to bandits; zero: no change.
- Average, median and mode of the peasant protection proportion
- Average, median and mode of the number of peasants a bandit preys upon[17]
- Number and percentage of peasants with the protection proportion defined by each bin (total bins defined by PROTECTION_PROPORTION_NUMBER_INTERVALS.

[17] Current implementation includes support for multiple bandits preying on multiple peasants; default is one bandit preys upon one peasant.

A log file is created for each scenario, containing the same statistics as defined in the summary file, for each period of the scenario.

Details

Initialization

Initialization for each scenario includes the following steps:

- Peasant population built: peasants numbering NUMBER_PEASANTS are created, with protection proportions randomly allocated at the boundaries of the bins numbering PROTECTION_PROPORTION_ NUMBER_INTERVALS, each bin spanning an interval of PROTECTION_PROPORTION_ INTERVAL_SIZE. Each peasant is given a contest function; all contest functions for a given scenario share the same γ, CONTEST_FUNCTION_GAMMA.
- Bandit population built: bandits numbering NUMBER_BANDITS are created.
- An Interaction Pattern is initialized with the bandit and peasant populations, with mode determined by flag NORMAL_INTERACTION_PATTERN (see Interactions).
- Statistics are initialized to zeros to collect observation data.
- Dynamics are added to the scenario. A DST dynamic is always added, with thresholds set per SURVIVE_THRESHOLD and THRIVE_THRESHOLD. A Role-Shifting dynamic may be added, if ROLE_SHIFTING is true.
- A pseudo-random number generator is initialized with the value of RANDOM_SEED
- An Equilibrium Seeker is initialized with the above objects. The run limit is set from RUN_LIMIT. The definition of equilibrium is set from EQUILIBRIUM_NUMBER_PERIODS_ WITHOUT_ADJUSTMENT and PAYOFF_DISCREPANCY_TOLERANCE (see Process Overview). The period is set to 1.

The Protection Model invokes the Equilibrium Seeker to begin execution.

Input

There are no input data; all execution is controlled by parameters (see Submodels)

Submodels

The routines of the Protection Model are controlled by the following parameters; details of their logic are outlined in the prior sections.[18]

- PROTECTION_PROPORTION_NUMBER_INTERVALS: positive integer number of bins into which Peasant protection proportions can be allocated
- PROTECTION_PROPORTION_INTERVAL_SIZE: size of each bin for Peasant protection proportions, $\in [0,1]$. Interval_size * number_intervals should equal 1.0.
- CONTEST_FUNCTION_GAMMA: input to the Peasant contest success function defining the defensive ability of the Peasant, $\in [0.5,1]$. Shared for all peasants in the population for a given scenario.
- ROLE_SHIFTING: determines whether the Role-Shifting dynamic will be invoked, $\in \{\text{true, false}\}$
- SURVIVE_THRESHOLD: determines the minimum payoff an agent must achieve in a period to have a single descendant in the next period, $\in [0,1]$
- THRIVE_THRESHOLD: determines the minimum payoff an agent must achieve in a period to have two descendants in the next period, $\in [0,1]$; greater than or equal to SURVIVE_THRESHOLD
- PAYOFF_DISCREPANCY_TOLERANCE: defines the maximum difference between the average payoffs for bandits and peasants for the two populations to be considered to be in equilibrium in the current period, $\in (0,1)$
- ADJUSTMENT_FACTOR_PERCENTAGE: percentage of the lower-performing role that will be shifted to the better-performing role at the conclusion of this period, if ROLE_SHIFTING is true, $\in (0,1)$
- EQUILIBRIUM_NUMBER_PERIODS_WITHOUT_ADJUSTMENT: positive integer number of consecutive periods without adjustment that must elapse for the scenario to be considered to be in equilibrium.
- NUMBER_PEASANTS: positive integer number of peasants that will be created during initialization

[18] Current implementation: there are other parameters in the v1.1 implementation that function, but have not been described in the studies that use the v1.1 code. Left at their defaults, they will not affect the function of the code. They implement: a transaction cost to bandits for preying on peasants; enable multiple bandits to prey upon multiple peasants; enable a matching function to determine the probability that a bandit is successful in preying on a target peasant.

- NUMBER_BANDITS: positive integer number of bandits that will be created during initialization
- RUN_LIMIT: positive integer number of periods that the scenario will execute without reaching equilibrium, before the scenario will stop.
- NORMAL_INTERACTION_PATTERN: determines whether bandits only interact with peasants, if true, or whether any agent can interact with any other agent, $\in$ {true, false}
- MAXIMUM_POPULATION_SIZE: positive integer setting the maximum size of the population of either bandits or peasants after replication; when exceeded, the scenario will stop after that period.
- NEW_PEASANT_GETS_BEST_PROTECTION_PROPORTION: positive integer setting the maximum size of the population of either bandits or peasants after replication; when exceeded, the scenario will stop after that period.
- FORCE_PEASANT_ALLOCATION_TO_HIGH_LOW: determines whether peasants will be initialized with one of two protection proportions, if true; $\in$ {true, false}. For use in evolutionary game theory analyses. Defaults to false.
- FORCE_PEASANT_ALLOCATION_LOW_INITIAL_PEASANTS: non-negative integer number of peasants less than or equal to NUMBER_PEASANTS that will be initialized with protection proportion FORCE_PEASANT_ALLOCATION_LOW_PROPORTION, if FORCE_PEASANT_ALLOCATION_TO_HIGH_LOW is true.
- FORCE_PEASANT_ALLOCATION_LOW_PROPORTION: if FORCE_PEASANT_ALLOCATION_TO_HIGH_LOW is true, sets the protection proportion $\in$ [0,1] of a number of peasants equaling FORCE_PEASANT_ALLOCATION_LOW_INITIAL_PEASANTS.
- FORCE_PEASANT_ALLOCATION_HIGH_PROPORTION: if FORCE_PEASANT_ALLOCATION_TO_HIGH_LOW is true, sets the protection proportion $\in$ [0,1] of a number of peasants equaling NUMBER_PEASANTS - FORCE_PEASANT_ALLOCATION_LOW_INITIAL_PEASANTS.
- RANDOM_SEED: long integer used as the seed for a pseudo-random number generator.

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