Endogenous Formation and Collapse Of Housing Bubbles

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Abstract

This paper develops an analytical framework and an agent-based spatial model of the housing market. We show that lenient financing, in particular, low down payment requirement has caused business cycles in the housing market. With little down payment, any fluctuations in the housing price will cause the collateral rate to fall below the threshold under which mortgage rate has to rise in response to a lower housing price. The agent-base model is based on our interviews with local real estate agents. The exploratory work in this paper will help us better understand the housing market, make policy advice, and prevent the damaging housing bubbles.

1 Introduction

The U.S. housing price bubble started around year 2000. At the height of the housing bubble, mortgage loans could be easily obtained on very lenient terms. Housing price has doubled in just a few years, and when it burst, price has dropped by nearly a third nation wide, and more than a half in the sand states (Arizona, California, Florida, and Nevada). The economy went into the most severe recession since the big depression in the 1930s .

What could have caused the damaging bubble? Some blame speculators; Others blame government sponsored entities like Freddie Mac and Fannie Mae for extending loan to riskier (sub-prime) lenders;

[1]We would like to thank Tom Randall Real Estate Team, Hunziker & Associates, and an anonymous realtor for their helpful comments and inputs.

Still others blame financial institutions for relentless lending. Many agreed that the bubble has to do with lenient financing. However, how exactly does lenient financing lead to a housing bubble remains unclear.

This paper proposes an analytical and an agent-based model to analyze the cause of a housing bubble. We found that lenient financing is responsible for causing price bubbles in the housing market. However, the main cause of the bubble is not low mortgage rate, the inclusion of buyers with low credibility, or speculative demand, as many have speculated. Even though those factors might have exacerbated the process. We believe that the main cause of the bubble is the low down payment requirement. The reason down payment plays such an important role in the formation of a housing bubble is that it determines the collateral rate (or equivalently the inverse of leverage) linked to the loan, which in turn will affect the riskiness of the loan and the mortgage rate.

2 Review of Literature

There are mainly three groups of papers in the housing literature. The first group focuses on speculative behavior in the housing market. Examples are Shiller[11] and Riddel [10]. The second group of papers, mostly done after the U.S. housing crises, look at the relationship between the housing market and sub-prime lending. Examples are Goetzmann et al. [5] and [1]. The third group studies land use choice and housing supply elasticities. Examples are Goodman et al. [7] and Glaeser et al [4]. However, according to a review by Mayer [8], the current literature is not satisfying in explaining and predicting a housing bubble. While the housing market makes up a significant part of national economy and household wealth, far less economic research has been done on the property market than on the stock market, the bond market, or the foreign-exchange market.

As for agent-based models, Goldstein [6] and Markose et al. [9] devised agent-based models for the interaction between the housing market and financial securities. Torrens [13] developed an multi-leveled agent based model for individual housing choices, but the model does not have a financial sector. The agent-based housing market model proposed by Geanakoplos et al (2012) [3] does have a sophisticated financial sector, but it focuses more on empirical validation of the model than analyzing possible causes of the housing bubble.

John Geanakoplos is among the first few to acknowledge the role leverage or equivalently, collateral rate plays in the business cycle. In his paper on leverage cycle [2], he looks at how a small shock in one sector can cause wide spread crises across sectors which payoffs are independent, because the most highly leveraged investors who are forced out of the market first happen to be the most optimal ones. In another paper by Thurner et al. [12], the authors argue that it is leverage that causes fat tails and clustered volatility. They use an agent-based model to show that even a small negative shock in the market will trigger large price drop, because leveraged investors are forced to sell the assets to stay within the maximum leverage rate.

Our paper also focuses on the collateral rate, which is the inverse of leverage. We are going to show how leverage or collateral in the housing market can cause regular business cycles. We will start with the analytical framework and then move on to the the agent-based model.

3 An Analytical Demonstration of Housing Bubbles

3.1 Collateral and Mortgage Rate

Collatal rate, c, is defined as the value of collateral per dollar of loan,

$$c \equiv \min\left\{\frac{V(\text{asset})}{V(\text{mortgage})}, 1+\bar{i}\right\} = \min\left\{\frac{1}{1-\frac{\text{down}}{\bar{p}}} + \frac{\dot{p}/\bar{p}}{\left(1-\frac{\text{down}}{\bar{p}}\right)}, 1+\bar{i}\right\} \tag{1}$$

where $\bar{p}$ is original purchase price, down is the amount of down payment required, and $\dot{p}$ is change in price. For simplicity, we ignore subsequent monthly payments and assume that the amount of loan equals purchase price minus down payment.We assume that perfect competition among funds for loan. Then non-arbitrage condition requires that the return on the mortgage loan equals an exogenous market return,

$$p^d(c)\cdot c + \left(1-p^d(c)\right)\cdot(1+r) = 1+\bar{i} \tag{2}$$

where r is mortgage rate, c is the collateral rate, $\bar{i}$ is the exogenous market return, and $p^d(c)$ is default probability of a lender, which is a decreasing function of collateral rate c. Eq 2 thus defines the relationship between mortgage rate r and collateral rate c. Take total differentiation with respect to c we get,

$$\frac{dr}{dc} = \frac{\frac{\partial p^d}{\partial c}\cdot(1+r-c) - p^d}{1-p^d\cdot(1+r-c)} < 0 \tag{3}$$

Mortgage rate increases as collateral rate decreases. Since $c = \min\left\{\frac{1}{1-\frac{\text{down}}{\bar{p}}} + \frac{\dot{p}/\bar{p}}{\left(1-\frac{\text{down}}{\bar{p}}\right)}, 1+\bar{i}\right\}$, $\frac{dr}{dc} < 0$ means $\frac{dr}{d\dot{p}} \leq 0$. There exists a negative relationship between mortgage rate and percentage change in price if the latter falls under the threshold, $\bar{i} - (1+\bar{i})\frac{\text{down}}{\bar{p}}$. The threshold is further away from zero with higher down payment requirement, and closer to zero with lower down payment

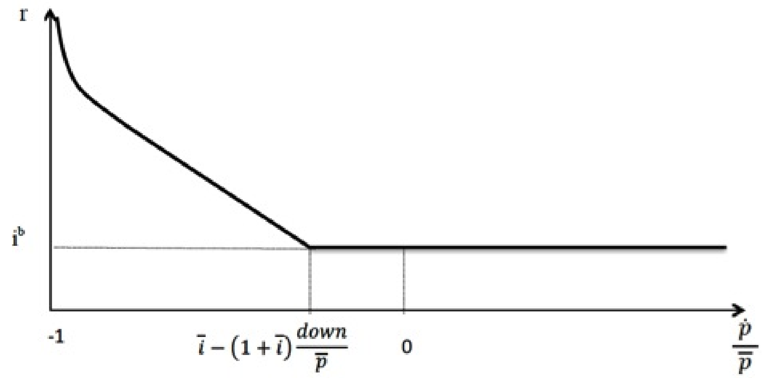

Figure 1: Mortgage Rate (r) and Collateral (c)

requirement. Fig 1 shows the relationship between mortgage rate and percentage change in housing price.

When a substantial amount of down payment is required, a decrease in housing price is unlikely to trigger an increase in mortgage rate. However when only little or none down payment is required, a small decrease in housing price will require the mortgage rate to increase to compensate for the shrink of value in collateral. An individual lender is regarded too small to affect the market price by raising mortgage rate. It rationally raises mortgage rate to protect itself from increased default risk. However, if every lender does so, demand for housing will be suppressed, which will in turn worsen the market condition and increase systematic risks.

3.2 Demand for Housing

Since the total cost of a house is $(1 + r) \cdot p$, non-speculative net housing demand, D^n, is decreasing in both r and p.

$$\frac{\partial D^n}{\partial p} < 0 \text{ and } \frac{\partial D^n}{\partial r} < 0 \tag{4}$$

Zero net demand therefore requires that mortgage rater r and housing price p move in the opposite direction.

Speculative net housing demand, D^s, is an increasing function of past housing price appreciation.

$$\frac{\partial D^s}{\partial \dot{p}} > 0 \tag{5}$$

When price is increasing, speculative net demand is positive; when price is stabilized, speculative net demand is zero; and when price is decreasing, speculative net demand is negative, meaning there is more supply than demand.

3.3 The Phase Diagram

Figure 2 shows the phase diagram of the system.

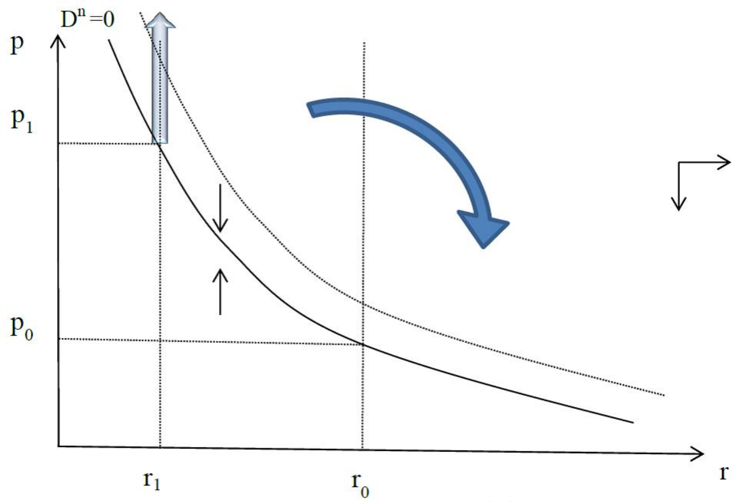

Figure 2: Phase Diagram of Housing Price (p) and Mortgage Rate (r)

The solid curve is the zero non-speculative net demand curve. When the exogenous market rate $\bar{i}$ decreases, as an response mortgage rate decrease from r_0 to r_1. Housing price starts to rise in response to the reduced mortgage rate. The increase of price does not stop at p_1, because there is positive speculative demand at p_1 since price has been increasing. Price will keep rising until total net demand (speculative and non-speculative) equals zero, $D^n + D^s = 0$. Then price will start to decrease because as price is stabilized, speculative net demand becomes zero but non-speculative demand is negative (above the solid curve). Price will keep decreasing and go below p_1 for the same reason price will go above p_1 when it is increasing: the existence of speculative demand. If we only have the above dynamics, housing price will fluctuate around p_1. The extend of the fluctuation depends on the relative size of speculative and non-speculative demand.

But there is a second, more substantial dynamics. Whether this second dynamics will take place depends on the level of down payment. The dashed line above the non-speculative zero net demand curve of $((1 + down) \cdot p, r)$, where (p, r) is the corresponding point on the non-speculative demand curve. A higher down payment means that the dashed curve is further away from the demand curve. Below the dashed curve, mortgage rate does not respond to housing price changes. Hence $\dot{r} = 0$. Above the dashed curve, however, mortgage rate would increase in response to decrease in housing price.

If down payment rate is low enough that speculative demand would drive the price above the dashed line (as is the case in Figure 2), the second dynamic is triggered and housing price will no longer fluctuate around p_1. Instead it collapses. Once in the region above the dashed curve, price will keep falling and mortgage rate keep rising until it falls under the dashed curve again. Once below the dashed line, mortgage rate will return to r_1, and the same cycle is repeated.

4 The Agent-based Model

One of the purposes of the agent-based model is to expand the basic dynamics in the analytical model to a richer representation of real-world scenarios. The housing market is very unique: it is less liquid, spatial, highly leveraged, and involves speculative behaviors. To this day, there are few models of housing market that take into account all of these complications. The agent-based model proposed in the paper could capture all the features mentioned above. It is able to model the housing market in a more realistic and rich setting.

To start, our housing market sits on a two dimensional landscape that contains multiple regions. Our general model has a landscape that is a five by five square. Each region has exogenous attributes called location quality, and endogenous attributes called neighborhood quality. There are five types of market participates: the real estate agent, the developer, buyers, homeowners, and the bank. We further distinguish buyers and homeowners as investors and non-investors. Investors buyers buy a property in hope of profiting from housing price appreciation. Regular buyers, on the other hand, obtain utility from living in the house. Figure 3 is a class diagram of the model. It outlines the model's class structure and demonstrates relationships between different types of agents.

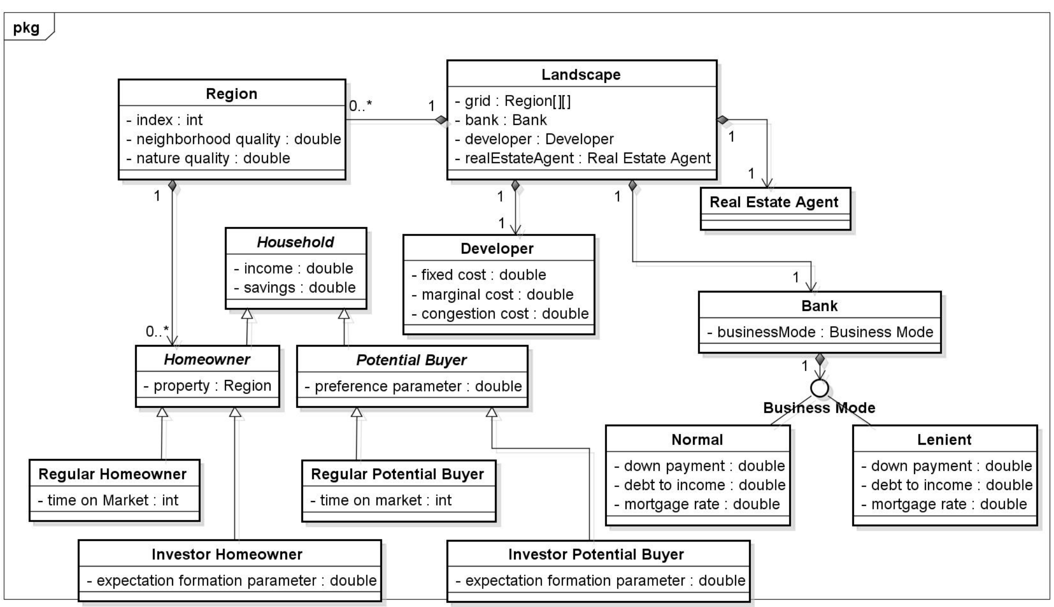

Figure 3: Class Diagram For The Housing Model

Each period in the model represents a month in real time. In each period, new buyers enter the market in search of a house. At the same time, existing homeowners decide whether to put their houses on the market for sale. Figure 4 is an activity diagram of the housing model. It illustrates how market participants interact. A red line represents an information flow, and a black line represents an action flow.

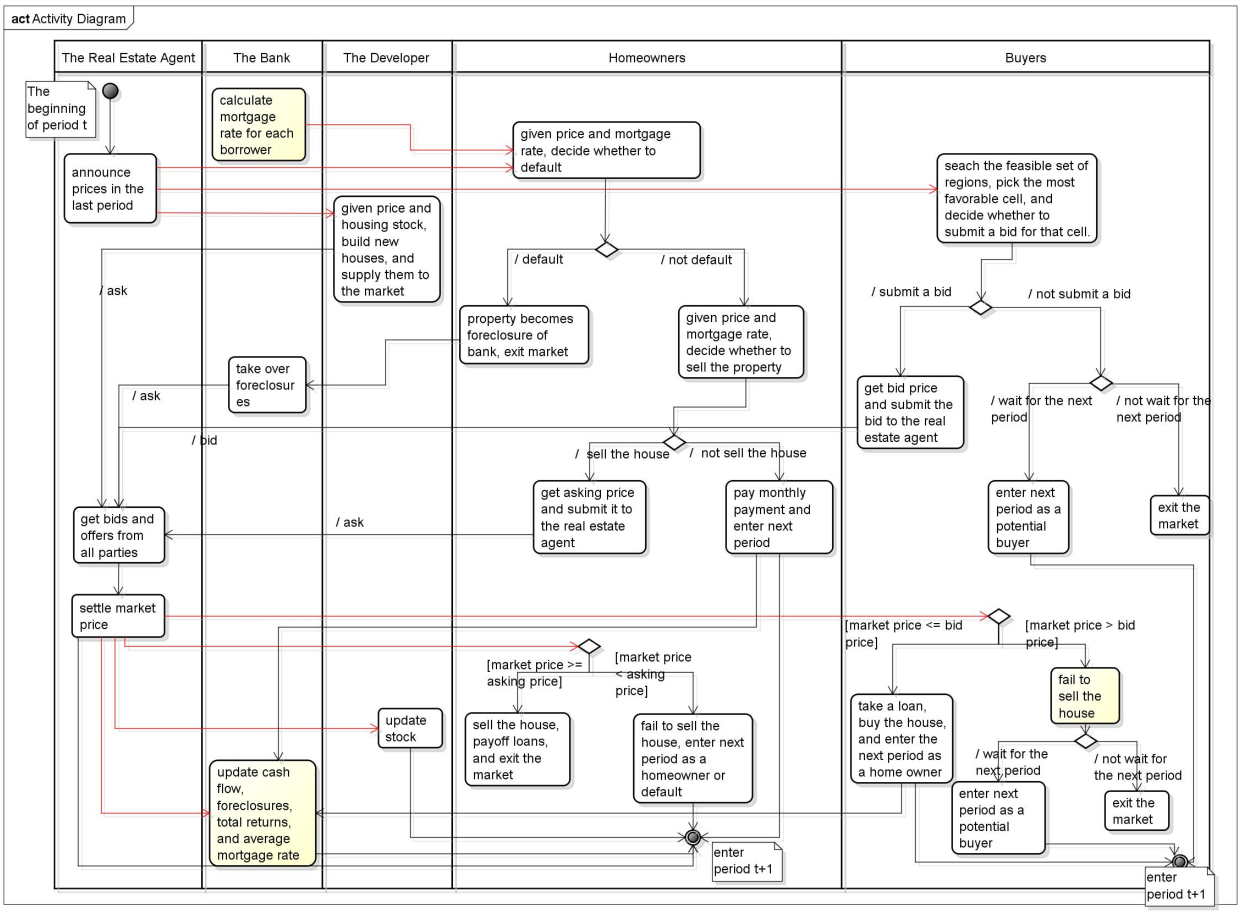

Figure 4: The Activity Diagram For The Housing Model

Summarized in a flow chart, the model works as follows:

Step 1 At the beginning of period t,

The Real Estate Agent: announces housing Prices for all regions in the previous period.

The Bank: announces mortgage rate and lending criteria.

Step 2 **The Developer**: given prices in the last period, builds new houses in each region.

A Homeowner: given price in the last period and mortgage terms, decides whether to default on the house. If chooses to default, the property goes into foreclosure and homeowner exits the market. Otherwise, chooses whether to sell the property. If chooses to sell, submits an asking price on the house. Otherwise, make monthly payment and enter period $t + 1$ as a homeowner.

A Buyer: searches a subset of all regions, submits a bid for am affordable house that gives her the highest utility (non-investor) or net return (investor).

Step 3 **The Real Estate Agent**: collects all the bids and asks, and settles the final market price for each region.

Step 4 **The Developer**: sells houses according to the market price. Unsold units become housing stock, and are put for sale in period $t + 1$.

The Bank: sells foreclosures according to the market price.

A Homeowner: if the asking price is lower than the market prices, sells her property and exits the market. Otherwise enter period $t + 1$ as a homeowner.

A Buyer: if her bidding price is higher than the market price, buys a house and enters period $t + 1$ as a homeowner. Otherwise choose whether to wait for another period or exit the market.

Step 5 End of period t. Enter period $t + 1$.

5 Results

Figure 5 shows the housing price with 20% and 10% down payment requirement.

(a) Home Price with 20 % down payment (b) Home Price with 10 % down payment

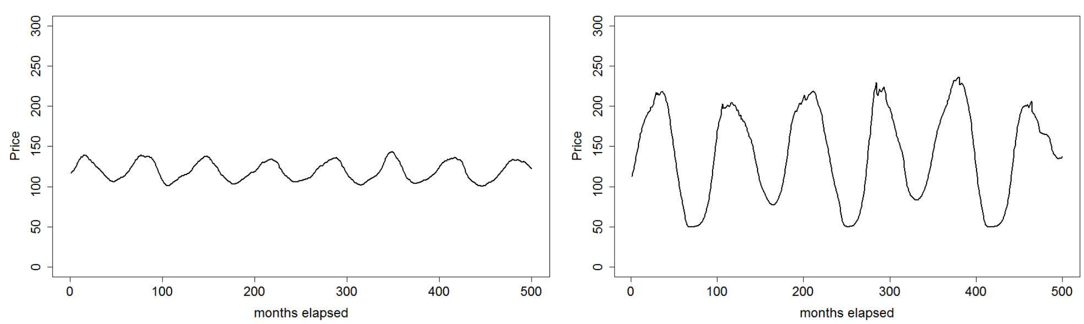

Figure 5: Housing Price Under 20% and 10% down payment

With 20% down payment requirement, housing price fluctuates slightly, probably due to the existence of speculative demand. While under 10 % down payment, price has large cycles that go from around 50 at the bottom to 250 at the peak. In other words, housing bubbles exist with 10% down payment requirement and

Figure 6 shows home price and foreclosure with 20% down payment and 10% down payment. If the price dives so low that the outstanding loan a homeowner owns is bigger than the value of the house plus a default cost (so the house is not only under water, but deeply under water), the homeowner will default. With 20% down payment requirement, foreclosure is very rare, while under 10% down payment requirement, foreclosure is rampant when the home price is falling quickly.

(a) foreclosures 20% down (b) foreclosures 10% down

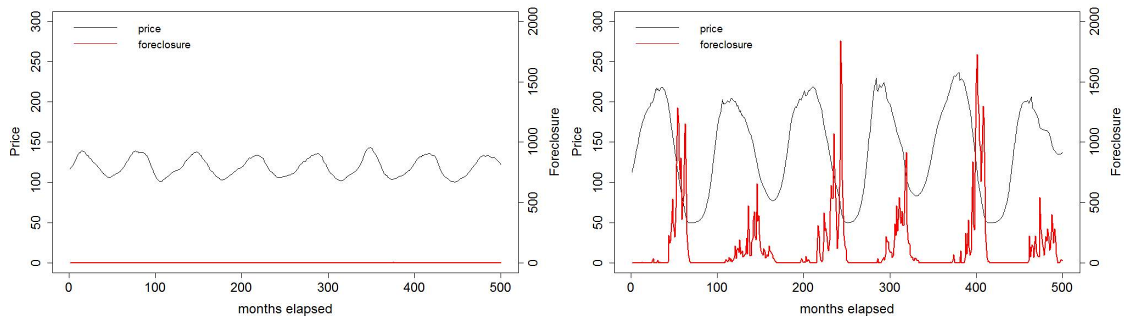

Figure 6: Housing Price and Foreclosure

Figure 7 shows home price and leverage with 20% down payment and 10% down payment. The leverage level, defined as outstanding loan over property price, equivalent of the inverse of collateral rate, moves in the opposite direction as housing price. Leverage increases when housing price decreases and decreases when housing price increases. Under 20% down payment rule, leverage is always below one, meaning there is always more asset than debt. Under 10% down payment rule, leverage is above one when the market collapses, meaning there is less asset than debt.

(a) leverage 20% down (b) leverage 10% down

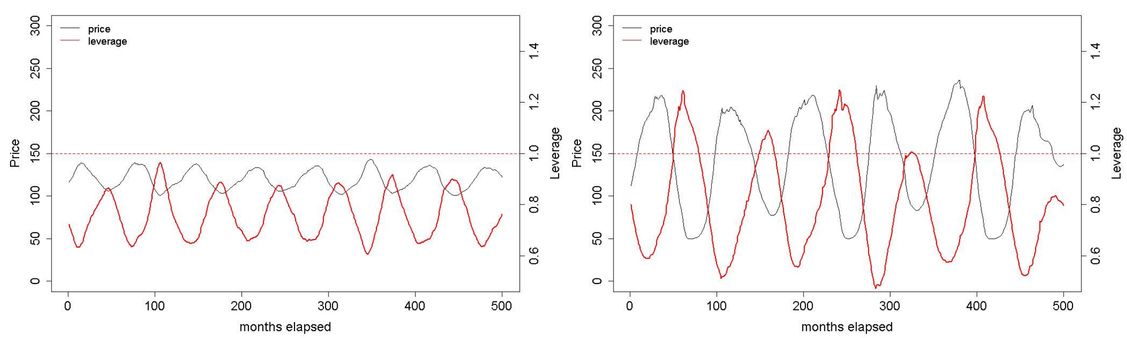

Figure 7: Housing Price and Leverage

Figure 8 shows home price and mortgage rate with 20% down payment and 10% down payment. Under 20% down payment, mortgage rate remains at baseline level of 2.5%. Under 10% down payment, mortgage rate remains at 2.5% when the housing price is stabilized or increasing. When housing price starts to fall, however, the rate jumps because the value of collateral has shrunk and the probability of default increases. Lenders no longer see mortgage loans as a safe collateral, and requires a higher rate to compensate for the increased risk. While an individual lender does so to protect herself from default risk, all lenders doing so will cause the price to dive deeper and the housing market to collapse.

(a) mortgage rate 20% down (b) mortgage rate 10% down

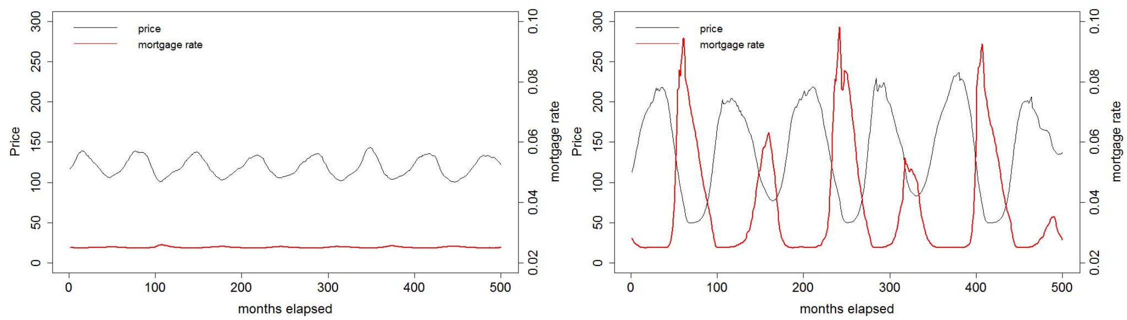

Figure 8: Housing Price and Mortgage Rate

Figure 9 shows home price and return on loans with 20% down payment and 10% down payment. Aggregate return on the loans rises when price rises, and it drops when price drops. However, there is a lag between the when housing price start to fall and when total return on loans start to shrink. It's not until the price has been falling for a while that total return starts to fall. Moreover, the total returns under 10% down payment rule is always higher than that under 20% down payment rule. Therefore it is rational for financial institutions to engage in lenient lending, even in the absence of securitization .

(a) total return on loans 20% down (b) total return on loans 10% down

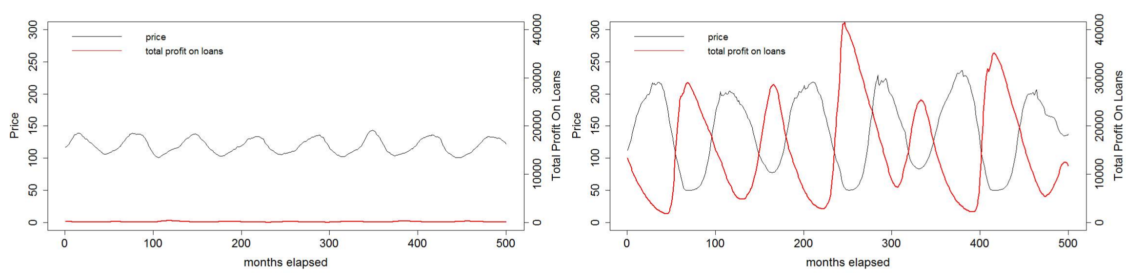

Figure 9: Housing Price and Total Return on Loans

6 Conclusions

This paper develops a analytical and an agent-based spatial model of the housing market. We have shown that the down payment requirement plays a key role in the formation of housing bubbles. With a small down payment, any fluctuations in the housing price will cause the collateral rate to fall below the threshold under which mortgage rate has to rise in response to the lower price. A higher mortgage rate will in turn suppress demand and kick the market into a downward spiral.

We incorporated in the agent-based model important aspects of the housing market that are largely ignored in the existing housing literature, such as the spatial aspect of the market, the role of time on market on buyer's decision making, and the inclusion of different types of buyers and homeowners. The model design is based on our interviews with real estate agents.

Simulation results show that low down payment requirement will cause housing bubbles. Under 10% down payment rate, housing price has experienced large bubbles; while under 20% down payment rate, housing price only fluctuates moderately. When housing price starts to fall under 10% down payment rate, foreclosures become wide-spread, leverage increases, mortgage rate increases, and total return on the loans decreases after a time lag.

After the crises, many people attribute the housing crises to aggressive lending and mortgage securitization by many financial institutions. Many argue that the twisted incentives caused by mortgage securitization has prevented the financial institutions from responding promptly to the first sign of a crises. Interestingly, our simulation results show that there is a time lag between the housing price starts to fall and the total return on loans starts to fall. It's not until the price has been falling for a while that total return on mortgage loans starts to fall. Therefore, even if the financial institutions are ready to respond to falling profits promptly, they will not prevent the bubble from collapsing.

Moreover, the total return on the loans under the low down payment requirement is about three times the return under high down payment requirement. Even after the housing bubble has burst, total return on loans with low down payment is much higher than that with high down payment. Hence there is nothing irrational for financial institutions to adopt a lenient lending criteria and expand lending.

To sum up, we have shown in both an analytical and an agent-base model that too little down payment will cause endogenous formation and collapse of a bubble in the housing market. Without irrational behavior from buyers and lenders, or external shock in the fundamentals, a housing bubble will arise when too little down payment is required. The reason is with little down payment, any fluctuations in the housing price will cause the mortgage rate to rise in response. A higher mortgage rate then further suppresses demand, turning the market into a downward spiral.

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Model Description: Overview, Design concepts, Details

1 purpose

The purpose of the model is to study the housing market and housing price bubbles. We want to see how aggregate market phenomenon such as housing bubbles can emerge from individual behavior such as household buying and selling houses, bank issuing loans, etc.

2 Entities, State Variables, and Scales

7 classes of agents.

- Real estate agent.
- Bank.
  - State: list of clients, mortgage rate.
  - Attributes: lending criteria, including maximum down payment rate and maximum debt to income ratio.
- Developer.
  - State: housing inventory in each region.
  - Attributes: construction costs, A, B and C.
- Homeowner.
  - State: property, time on market, outstanding loans, time since purchase.
  - Attributes: income, default cost.
  - The class Homeowner has two sub-classes:
    - Investor Homeowner.
      - Attributes: expectation formation parameter, w.
    - Non-investor Homeowner.
      - Attributes: utility formation parameter, α, β, and γ.
- Buyer.
  - State: time on market.
  - Attributes: income, savings
  - The class Buyer has two sub-classes:

        * Investor Buyer.
            · Attributes: expectation formation parameter, w.
        * Non-investor Buyerr.
            · Attributes: utility formation parameter, α, β, and γ.

- Region.
  - State: housing price, list of residents, number of foreclosures, neighborhood quality.
  - Attributes: index, list of neighboring regions, location quality
- Landscape.
  - State: total population, total foreclosure.
  - Attributes: list of regions contained in the landscape.

3 Process Overview and Scheduling

Time is modeled as discrete steps in the model. One period corresponds to one month in real time. At the beginning of period t:

1. developer builds new houses: Algorithm 3.1
   Algorithm 3.1: DEVELOPER BUILDS NEW HOUSES ()

   for each *region* $\in$ *landscape*
   do {
   get price history in that region
   if profitable to build new houses
   then build new houses in that region which will take $\bar{t}$ periods to finish
   put newly finished houses started $\bar{t}$ periods ago on the market
   put unsold houses from previous periods on the market
   }

2. homeowners update states: Algorithm 3.2
   Algorithm 3.2: HOMEOWNER UPDATE STATES()

   for each region r $\in$ landscape
   do for each homeowner i $\in$ region r's homeowner list
   do {
   time since purchase++
   update outstanding loan
   update whether to dafault
   if decide to default
   then default, leave the system, and lender takes over property
   else {
   pay monthly payment and decide whether to sell the property
   if decide to sell the property
   then calculate ask and submit the ask to the real estate agent
   else hold the property and go into period t+1
   }
   }

3. new buyers are generated: Algorithm 3.3

Algorithm 3.3: GENERATE NEW BUYERS(n_1, n_2, meanIncome$_1$, meanIncome$_2$, sdIncome$_1$, sdIncome$_2$)

for $i = 0$ **to** n_1
  do create buyer i: type=non-investor, income = N_+ (meanIncome$_1$, sdIncome$_1$)
for $j = 0$ **to** n_2
  do create buyer j: type=investor, income = N_+ (meanIncome$_2$, sdIncome$_2$)

4. new and existing buyers search for properties: Algorithm 3.4

Algorithm 3.4: BUYER SEARCH()

for each buyer i ∈ all buyer list
  do
    for each region r ∈ buyer i's choice set of regions
      do calculate utility or return of that region
         update maximum utility or return if necessary
    if maximum utility or return is higher than zero
      then put in a bid for the chosen region
      else not submit a bid

5. real estate agent settles price: Algorithm 3.5

Algorithm 3.5: REAL ESTATE AGENT SETTLES PRICE ()

order bids from high to low
order asks from low to high
settles market price accordingly

6. buyers update states: Algorithm 3.6

Algorithm 3.6: BUYERS UPDATE STATES()

for each buyer i ∈ all buyer list
  do
    if has submitted a bid & bid $\geq marketprice$
      then buyer i buys the house
           buyer i becomes a homeowner
           buyer i is added to the resident list in the chosen region
           buyer i takes out a loan and is added to the lender's client list
      else if decide to stay for another period
    time on market + +
      else exit the market

7. homeowner sellers update states: Algorithm 3.7

   Algorithm 3.7: SELLERS UPDATE STATES()

   for each homeowner i $\in$ all homeowner list

   do
   - **if** has submitted an ask & ask $\leq$ *marketprice*
     - **then**
       - homeowner i sells the house
       - **if** selling proceeds $\geq$ outstanding loanhomeowner i fully pays off loan
       - **else** homeowner i pays the lender the selling proceeds
   - **else** hold the property and enter the next periods as a homeowner

8. developer updates inventory: Algorithm 3.8

   Algorithm 3.8: DEVELOPER UPDATES INVENTORY()

   for each *region* $\in$ *landscape*

   do
   - get number of houses sold in last period in that region
   - new inventory = last inventory + new construction − number of houses sold last period

9. lender updates states: Algorithm 3.9

   Algorithm 3.9: LENDER UPDATES STATES()

   for each foreclosure $\in$ foreclosure list

   do
   - **if** ask $\leq$ *marketprice*
     - **then**
       - sell the foreclosure property
       - remove the property from foreclosure list
   - update average mortgage rate
   - update average return on loans

10. regions update states: Algorithm 3.10

    Algorithm 3.10: REGIONS UPDATE STATES()

    for each *region* $\in$ *landscape*

    do
    - add new price to price history
    - update mean household's income in that region
    - update number of foreclosures in that region
    - update neighborhood index

4 Design Concepts

4.1 Basic Principles

At the submodel level, consumer theory (non-investor), producer theory (developer), market arbitrage condition (lender), and the risk-neutral expectation formation (investor) are used.

4.2 Emergence

The housing price, population, neighborhood index, mortgage rate, total number of foreclosures, and total return on loans are modeled as emerging from the individuals buyers, sellers, lenders, and borrowers.

4.3 Adaptation

All agent types in my model change their trading behavior in response to changes in the housing market conditions. Investor households, for example, predict future price from past prices and tries to make the best out of it. Non-investor households tries to obtain the highest utility by choosing where to live, given current market price and neighborhood quality.

4.4 Objectives

The objective of an investor household is the expected return on their housing purchase. The objective of a non-investor household is the level of happiness measured or utility. The objective of a developer is total profit. The objective of a lender is total return on the loans.

4.5 Learning

No, my agents does not change their behavior rules over time.

4.6 Prediction

For investors, predictions of future price is a function of past price.

4.7 Sensing

Housing price and neighborhood quality are (gloabally) public information that all agents can sense or observe. Actions of other buyers and sellers in the market are not observable for an individual buyer or seller, and is only observable to the real estate agent. Lenders also have information of its clients. But a client does not have information about other clients. There is no social network in terms of information dissipation.

4.8 Interaction

The following interactions are indirect: Buyers and sellers interact with each other through market price. Homeowners interact with each other through neighborhood quality. Borrowers interact with each other through mortgage rate. Homeowners default on the loans, resulting in lenders taking over the property. The following interactions are direct: buyers and sellers submit bids and asks to the real estate agent; borrowers borrow from lenders who charges a mortgage rate for the loans. Households move into a region.

4.9 Stochasticity

Household's income is assumed to be a random variable from a normal distribution. Since the generation of income from the labor market is outside the scope of this paper, random income is used to reproduce variability in the income generation processes for which it is unimportant to model the actual causes of the variability in income.

4.10 Collectives

Individuals in my model do not belong to aggregations.

4.11 Observation

Housing price, number of foreclosures, leverage, mortgage rate, total returns on loans, number of homeonwers, and so on.

5 Initialization

The initialization and model calibration is listed in Table 1.

Table 1: Initialization and Model Calibration

Variable	Symbol	Type	Value
number of regions		I_+	25 (5X5)
initial housing price		R_+	$N_+(100, 5)$
household i's income	m_i	r_+	$N_+(3, 1.5)$
household i's savings	ω_i	r_+	$6 \cdot m_i$
construction cost part i	A	R_+	0.0
construction cost part ii	B	R_+	1.0
construction cost part iii	C	R_+	0.25
investor j's weight on neighborhood price	w_j	R_+	$U[0, 1]$
non-investor i's weight on leverage	α_i	[0,1]	$U[0, 1]$
non-investor i's weight on neighborhood	β_i	[0,1]	$U[0, 1]$
non-investor i's weight on nature	γ_i	[0,1]	$U[0, 1]$
non-investor homeowner's probability to sell		R_+	0.05
non-investor buyers as % of non-investor homeowners		R_+	0.05
investor buyers as percentage of total homeowners		R_+	0.05
homeowner i's default cost		R_+	$12 \cdot m_i$

In this first-cut model, the initial values are chosen arbitrarily and not from real data. However, in the second-cut model in which we are going to study a specific housing market, initial values will be derived from data. The initial values are allowed to vary. Multiple simulations have been run with different initial values to ensure the robustness of model results.

6 Input data

In this first-cut model, we do not use input from external sources. However, in the second-cut model in which we are going to study a specific housing market, we are going to use data files as input to represent processes such as income generation process that is outside the scope of the model and changes over time.

7 Submodels

7.1 The Mortgage Contract

A mortgage contract is made up of the following components,

- eligibility requirements:

  - maximum debt to income ratio, $\overline{lev}$
  - minimum down payment rate, $\underline{\text{down}}$
- value of loan, L_0
- loan duration in months, T
- mortgage rate, r
- collateral, a property in region g

$mp\left(L, r, T^l\right)$ is monthly payment with loan value L, rate r, and remaining duration or time left T^l, which is calculated in the following equation,

$$\left(1+\frac{r}{12}\right)L = \sum_{t=0}^{T^l} \frac{mp}{(1+\frac{r}{12})^t} \tag{1}$$

$$\Rightarrow mp(L, r, T^l) = \frac{\frac{r}{12}\cdot L_0}{1-\frac{r}{12}^{T^l}} \tag{2}$$

Outstanding loan at time t, L_t, is defined in the following equation,

$$L_0 = (1-\underline{\text{down}})\cdot \bar{p} \tag{3}$$

$$L_t = \left(1+\frac{r}{12}\right)\cdot (L_{t-1} - mp(L_{t-1}, r, T-t-1)) \tag{4}$$

where $\bar{p}$ is the purchase price.

Household i is eligible for a mortgage contract if and only if all of the following conditions are met

$$\bar{p} - L_0 = \bar{p}\cdot(1-\underline{\text{down}}) \leq \omega_i \tag{5}$$

$$lev_i \equiv \frac{mp\left(L_0, r, T\right)}{m_i} \leq \overline{lev} \tag{6}$$

where m_i is household i's monthly income and ω_i is household i's savings.

7.2 Lender

We assume zero expected net return on the loans, due to perfect competition among funds for the loans. The market arbitrage condition is the expected return on the loans equals the exogenous market rate, $\bar{i}$,

$$p_t^d \cdot c_t + (1-p_t^d)\cdot(1+r_t) = 1+\bar{i} \tag{7}$$

where p_t^d is the probability that the household default on the loan.

p_t^d is the ratio between total number of defaults in the last $\bar{t}$ periods and the total number of loans in the last $\bar{t}$ periods,

$$p_t^d = \frac{\sum_{b \in B} 1(\text{borrower b went into foreclosure })}{\sum_{b \in B} 1} \tag{8}$$

where B is the set of borrowers who have taken out a mortgage loan in the last $\bar{t}$ periods.c_t is the mean value to loan ratio at time t. c_t is defined in the following equation,

$$c_t = \frac{\sum_{b \in B} c_t^b}{\sum_{b \in B} 1} \tag{9}$$

where B is the set of borrowers who have taken out a mortgage loan in the last $\bar{t}$ periods.c_t^b is collateral rate for borrower b, defined as,

$$c_t^b = \frac{p_t^i}{L_b^i} \tag{10}$$

Mortgage rate at period t, r_t thus is,

$$r_t = \frac{1 + \bar{i} - p_t^d \cdot c_t}{1 - p_t^d} - 1 \tag{11}$$

7.3 Non-investor Buyer

For non-investor household i, its objective is,

$$\max_{g \in F^i} \quad U^i \left(\text{nbhd}^g, \text{loc}^g, (1 - lev_i^g)\right) \tag{12}$$

where F^i is the feasible choice set of regions for household i, including the option of not buying a house or g=null. Assume U(null)=0. s_i is household i's savings, and lev^g is the household's leverage if it buys a house in region g, formally,

$$lev_i^g \equiv \frac{mp\left(p^g \cdot (1 - \text{down}), r^i\right)}{m^i} \tag{13}$$

where mp is monthly payment and m^i is monthly income of household i.A non-investor buyer i's action at period t, a_t^i, is,

$$a_t^i = \begin{cases} \text{bid}_t^i = (1 - \text{margin} + \delta \cdot \text{TOM}_i) \cdot P_{t-1}^i & \text{if } \text{TOM}_i < T \\ \text{exit market} & \text{if } \text{TOM}_i \geq T \end{cases} \tag{14}$$

where $g^*(i)$ is the region chosen by household i at time t, TOM_i is household i's time on market, margin is a buyer's targeted discount on current price, δ is the percentage increase in price each period the buyer's bid is not accepted, and P_{t-1}^i is the last period price in the household i's chosen region. Once a non-investor buyer has picked up the region that gives her the highest utility in her choice set, she then puts a bid for a house in that region.A non-investor buyer's bidding price depends on the number of periods she has been on the market. The longer the buyer has been on the market, the more impatient she is, and the higher the bidding price. A buyer has a maximum waiting time of T periods. A buyer will continue to increase her bid until the time she has been on the market exceeds her maximum waiting period ($t \geq T$).

7.4 Investor Buyer

We define I_t^g as the information set in region g at period t. I_t^g includes the following information,

$$I_t^g(M) = \begin{bmatrix} p_t^g & p_{t-1}^g & \cdots & p_{t-M}^g \\ p_t^{g_1} & p_{t-1}^{g_1} & \cdots & p_{t-M}^{g_1} \\ & & . & \\ & & . & \\ & & . & \\ p_t^{g_N} & p_{t-1}^{g_N} & \cdots & p_{t-M}^{g_N} \end{bmatrix} \tag{15}$$

where $\{g_1, g_2, ..., g_N\}$ is the index for region g's neighboring regions, and M is the maximum length of memory an investor has about prices in the past.$ER^j(g)$ is investor buyer j's expected net return in region g. Assume $ER^j(\text{null}) = 0$. $ER^j(g)$ for $g \neq null$ is defined as follows,

$$ER_t^j(g) = F\left(I_t^g; w_j\right) \tag{16}$$

The parameter w_j is the weight investor j put on the region itself relative to its neighboring regions in the formation of expected returns. The expected net return also depends on the mortgage rate, r, and the cost of keeping a house (before the purchase is made), $cost^b$, which includes property tax, transaction cost, depreciation, and (minus) rental income.Investor j's expected return on a housing investment in region g depends on prices in region g in the past. It also depends on prices in region g's neighboring regions in the past. For investor household j, its objective is,

$$\max_{g \in F^j} \quad ER^j(g) \tag{17}$$

where F^j is the feasible choice set of regions for household j, including the option of not buying a house or g=null.Investor buyer j's action at period t is,

$$a_t^j = \begin{cases} \text{bid}_t^j = \left(1 + \frac{R^j(g)}{12}\right) \cdot P_{t-1}^j & \text{if } R^j > 0 \\ \text{exit market} & \text{if } R^j \leq 0 \end{cases} \tag{18}$$

where P_{t-1}^j is price in the region chosen by household j, and R^j is investor j's expected net return on its chosen region.

7.5 Non-investor Homeowner

A non-investor homeowner i's action at period t, a_t^i, is,

$$a_t^i = \begin{cases} \text{hold property} & \text{if } p^d \geq \overline{p^d} \\ \text{ask}_t^i(i) = (1 + \text{margin} - \delta \cdot \text{TOM}_i) \cdot P_{t-1}^i & \text{if } p < \overline{p^d} \\ \text{go into foreclosure} & \text{if } P_{t-1}^i < V(\text{loan}) - \text{default cost} \end{cases} \tag{19}$$

where p^d is the random number uniformly distributed between zero and one, and P_{t-1}^i is the price in the region where household i' owns property, and $\overline{p^d}$ is the exogenous probability the household needs to sell the property. TOM_i is household i's time on market, margin is a seller's targeted margin over current price, δ is the percentage decrease in price each period the buyer's bid is not accepted, and T is the maximum number of time on market.

7.6 Investor Homeowner

For investor household j, its objective is,

$$\max_{\text{hold,sell}} max\left\{ER^j, 0\right\} \tag{20}$$

where ER_t^j is investor buyer j's expected net return on the its own property at time t.

$$\begin{gathered} ER^j = ER^j\left(p_{t-l}^g, p_{t-l}^k, r, \text{cost}^a; \omega_j\right) \\ \forall k \in N^g \text{ and } \forall l = 1, 2, ..., L \end{gathered} \tag{21}$$

It is the same as in Eq 16 except that here we use the cost after purchase, cost^a instead of cost before purchase, cost^a as in Eq 16. Cost such as transaction cost is sunk once the purchase has been made, hence is not counted in the cost after purchase.Investor buyer j's action at period t is,

$$a_t^j = \begin{cases} \text{hold the property} & \text{if } R^j(g) \geq 0 \\ \text{ask}_t^j = \left(1 + \frac{R^j}{12}\right) \cdot P_{t-1}^g & \text{if } R^j < 0 \text{ and } P_{t-1}^g \geq V(\text{loan}) - \text{default cost} \\ \text{go into foreclosure} & \text{if } P_{t-1}^j < V(\text{loan}) - \text{default cost} \end{cases} \tag{22}$$

where default cost is the cost associated with default, such as legal cost, loss of credibility, and emotional cost.

7.7 Developer

$$TC_t^r(h) = Ah + \frac{B}{2}h^2 + \frac{C}{2}\left(\bar{H}_t^r\right)^2 \leftrightarrow MC_t^r(h) = A + Bh + C\bar{H}_t^r \tag{23}$$

where $\overline{H}_t^r$ is total number houses already built in region r at period t, and h is the number of houses built in a single period. A, B, and C are positive numbers. We assume that the more houses exist in the region, the more expensive to build another one, due to land scarcity, regulations, etc.

The developer's supply in region r in period t, h_t^r, is set where price equals marginal cost (in $\bar{t}$ periods ago): $P_{t-\bar{t}}^r = A + Bh_t^r + C\bar{H}_{t-\bar{t}}^r$, where $\bar{t}$ is the number of periods it takes for the developer to build houses. Hence h_t^r equals,

$$h_t^r = \frac{P_{t-\bar{t}}^r - A - C\bar{H}_{t-\bar{t}}^r}{B} - \mathrm{stock}_t^r \tag{24}$$

where stock_t^r is housing stock in region r at period t. stock_t^r is updated at the end of each period,

$$\mathrm{stock}_t^r = \mathrm{stock}_{t-1}^r + h_t^r - \mathrm{sold}_t^r \tag{25}$$

where sold_t^r is the number of houses sold in region r during period t.

The asking price for a newly built house i, ask_i, is its marginal construction cost,

$$\mathrm{ask}_i = A + C\bar{H}_{t-\bar{t}}^r + Bi \tag{26}$$

$\forall i = 1, 2, ..., h_t^r$.Houseing stocks are sold at $A + C\bar{H}_t^r$ in period t.