Noname manuscript No.
(will be inserted by the editor)

A dynamic equilibrium model of how regulative and normative institutions influence economic behavior and growth.

Walid F. Nasrallah · Karim A. Cheaib · Ali A. Yassine

Received: date / Accepted: date

Abstract We present a non-cooperative game theoretic model with few parameters that combines three strands of economic research - institutions and welfare economics, crime and punishment, and kindness and altruism. We use this model to investigate how pre-existing institutions foster the formation of markets, as described by Douglas North's investigations of how societies diverge from the efficient state predicted by classical micro-economics.

Our "theta-pi" model places market participants within institutions, as described by North, but without violating or expanding rational decision theory. The inputs to the model are the strength and reach of different types of institutions, specifically ones that coerce with threat of punishments and ones that change perceptions of what is good and right and desirable. The outputs are the benefits available to various parties purely as a result of rational actions in response to these institutions. The value added by different types of institution varies with the types of economic transactions that are technologically possible. This paper examines a few results of this model by way of proof of concept, but we hope that the explanatory flexibility of this model will yield many further investigations into into the growth of human societies.

Keywords Rational Decision Model · Classes of Institutions · Trust · Empathy · Coercion

W. F. Nasrallah · A. A. Yassine
American University of Beirut, Beirut, Lebanon
Tel.: +961 1 350000
E-mail: walid@alum.mit.edu

A.A. Yassine
E-mail: ay11@aub.edu.lb

K. A. Cheaib
E-mail: kac2177@caa.columbia.edu

1 Introduction

The role of pre-existing institutions in the formation of a market for economic transactions has been getting a good deal of research attention. There is a widespread realization that any economic transaction, no matter how profitable, will only occur when all parties to the transaction can overcome their fear that any other party will use violence, deception, or other nefarious means to abscond with more of the value than was initially agreed (Selznick, 1994; Sen, 1997; Olson, 1984; Olson and Kähkönen, 2000; Greif, 2006; Scott, 2007; Ebner and Beck, 2008; North, 1990; North et al, 2009; Acemoglu and Robinson, 2012). This recognition, shared by researchers in many different fields, has yet to challenge the elegance and ubiquity of the canonical microeconomic models of Adam Smith and Léon Walras despite many calls to do so (Keen, 2003). Instead, the field of development economics has relied on empirical correlations between measures of institutional quality and measures of economic development (Chong and Calderón, 2000).

What follows is an attempt to build a simple quantitative model that abstracts some of what we know about institutions to a level of generality similar to the Boolean representation of logic as a simple choice between true=1 and false=0. This model will be used to investigate the effects institutions, abstractly represented, on rational micro-economic decision making. The ultimate goal is to derive patterns from the analysis of the model that can explain historical observations about economic development under imperfect governance. For example, institutions can spur economic growth, but can also become a hindrance – what causes the change? A party that gets the role of maintaining an institution can afford to give itself certain privileges and exemptions – when is doing so tolerable and when does it start to adversely affect the society's success?

1.1 Three types of institutions

Scott (2007) proposed a three-way system to both classify and analyze institutions. While acknowledging that every institution really has elements of all three members of Scott's typology, we will simplify by focusing on the classification interpretation in this paper. Scott distinguishes between:

1. coercive / regulative / retaliative institutions
   that deal with establishing legal guilt or innocence;
2. empathic / moral / normative / social-obligation
   institutions that distinguish shame and honor; and
3. cognitive / cultural / informative / mimetic
   institutions that operate on the level of knowledge versus ignorance (or certainty versus confusion.)

Scott (2007) explains how a memory of a past pattern of action can lead to a preference for following this pattern in future. This pattern becomes an

institution only when many people share a reasonable expectation that *others* will also follow this pattern in future. He also distinguishes between the initial need that calls for an institution and the needs of "institutional entrepreneurs" who keep the institution from decaying. This is all well accepted by the authors in various fields cited above. We are interested in using this typology to capture with some rigor the interaction between the institution and the basic unit of economic activity, the trading pair. We take the three in reverse order of relevance to our modeling effort.

1.1.1 Cognitive Institutions

The first type of institution is the cognitive or informational. Cognitive institutions facilitate trade by ensuring that different parties attach the same meanings to different symbols and situations. They include language, unified weights and measures, monetary instruments, notions of possession and legitimate ownership, and measures of the quality of goods. Cognitive institutions have a peculiar property that they are self-perpetuating. It is possible to gain the benefits of a cognitive institution by observing it in action. We therefore neglect cognitive institutions in this model because, in the equilibrium state that we wish to describe, the passage of time will have ensured that all who stand to benefit from a cognitive institution have the means to obtain that benefit.[1]

1.1.2 Coercive Institutions

The next type of institution classified by Scott (2007) has to do with laws, law enforcement and punishment or retribution. We make a careful distinction between the actions of an institutionalized law enforcer and the the usual permutations of individual retaliatory behavior, as described in the traditional literature on Game Theory, specifically the Iterated Prisoners' Dilemma (IPD) (Maynard Smith and Price, 1973; Axelrod, 1980, 1987; Aktipis, 2004). Instead of the rational benevolent dictator proposed by Cohen et al (2001), we can simply add an impersonal "probability of getting caught" to the decision parameters, and call it π (pi) (see Section 3.2).

1.1.3 Empathic Institutions

Institutions such as the family, the tribe, and different ideologies that stress belonging to a more universal group can all induce people to engage in economic transaction that they would normally fear. We propose to model the effects of the increased trust engendered by these institutions using a single variable, θ (theta), that describes a limited *expansion of the locus of utility.*

[1] An exception to this is when the equilibrium state itself is path-dependent. When a different equilibrium is reached depending on what was assumed early on, then we have a self-fulfilling cognitive effect. We revisit this in Section 3.

Without delving into cooperative rationales as classified by Sachs et al (2004), we assume that some actors, some of the time, act as if they are maximizing the utility that accrues to themselves plus a small fraction θ of the utility that accrues to the counter-party in the transaction[2]. We call this abstraction the "empathic institution" or "theta-institution", and describe it below in Section 3.1.

1.2 Synopsis

In the next section, we describe some of the prior work that has inspired this model. We then (in Section 3) build our quantitative model based on the assumptions described above. Section 4 gives some quantitative outcomes of letting **theta** and **pi** institutions simultaneously act on the same population. Section 5 summarizes the most significant ramifications of the analysis. Finally Section 6 summarizes the contributions, limitations, and expected future lines of research that might build on the model's results.

2 Background

The connection between institutions and prosperity has been explored by scholars in different fields, from sociology to history to economics. Three men who themselves conducted encyclopedic surveys as well as making recognized advances in the state of the art are Douglas North (North, 1990; North et al, 2009), Mancur Olson (Olson, 1971, 1984; Olson and Kähkönen, 2000; Olson, 2000) and Dick Scott (Scott, 2007). To this large field, we are proposing a small addition of a simple quantitative model that can be configured to represent different social states, and then used to draw non-intuitive conclusions about things like state transitions that occur at certain points in the development of institutions.

Olson (1971) expands on the story of the "stationary bandit" who makes violence into a special type of useful institution, "the state", and explains how the usefulness of the state to economic growth can ebb and flow in cycles. This view is expanded in light of economic variations across recent history and geography in several other works (Olson, 1984, 2000; Olson and Kähkönen, 2000)

North (1990) expanded into book form several notions about how institutions foster wealth creation. Institutions facilitate economic interactions by reducing uncertainty, and the organizations that maintain an institution are

[2] This might come about for many reasons, such as sharing and believing a common mythology, listening to "love thy neighbor" sermons, reacting to social cues, analyzing, either cognitively or by natural selection, a series of past transactions that led to success or failure, or anticipating, either cognitively or emotionally, a series of future transactions with similar partners. The pattern of behavior might entail reducing the harm one causes to counter-parties (integrity), or self-identifying with others' misfortunes regardless of one's role (altruism).

distinct from the institution itself and can themselves be economic actors in competition with non-institutional actors. He employs these insights to explain several historical phases in the growth of the world's economy, and concludes by noting that the serious study on institutions is just beginning.

North et al (2009) focuses on the protection against violence that institutions, especially of the regulative type, provide. The "natural state" describes multiple sovereign sets of institutions that coexist with their neighbors through a mutual agreement to maintain each other's monopoly not only on violence but also on legitimacy. This is contrasted to the "open state" when all economic actors have equal rights and sovereignty is maintained through different arrangements. North et al (2009) concludes that a society that achieves the "open state" will have a higher level of economic performance, and offers economic development data to support this. It is not clear that events such as the global economic crisis of 2008 are not better explained with a more cyclical paradigm such as the one proposed by Olson (2000).

Another body of research, notably inspired by Becker (1968), focuses on the consequences of crime on economic activity. Polinsky and Shavell (1999b,a) more explicitly modeled various elements in the economics of public law enforcement and explained their consequences on social welfare. The research presented in this paper will focus on the wider interpretation of law enforcement as an institution, leaving behind of specificity usually obtained with a larger number of parameters.

The "Prisoners' Dilemma" started as a mathematical curiosity in the works of (Von Neumann and Morgenstern, 1967), and has had a long and storied history as the building block of many models of interactions in fields ranging from biology (Maynard Smith and Price, 1973) to political economy (Bueno De Mesquita et al, 1999). Especially significant in the understanding of the game's dynamics has been the demonstration that simpler and more generous strategies out-compete more complex and selfish ones (Axelrod, 1980, 1987; Nowak and Sigmund, 1993). Other games, such as the "Dictator" and the "Anti-Prisoners'-Dilemma" game, have also been used as bases for squaring altruism with self-interest (Neilson, 2009). Of particular interest in the construction of our model is the effect of allowing a choice to cease participating in the game as a strategy to both avoid loss and to inflict damage (Aktipis, 2004).

Game theory is not the only source of rational explanations ofr altruistic behavior. Inspired by psychological and biological research, social scientists have devised several models of how altruism can survive for reasons such as human docility (Simon, 1990). Some underlying principles, such as reciprocity and equity, appear repeatedly. The principle that people are not only motivated by their own payoff but also by their relative payoff to the opposing party, for example, has been shown to be consistent and effective in explaining various cases (Bolton and Ockenfels, 2000).

Both game theory and institutions have a much longer and more varied history than the above brief tour implies. However, we can now summarize what we need to proceed with model development. Earlier research in both

veins was more concerned with why different social structures (e.g. third-world economies and various historical (non-cooperative) "dark ages") are slower to develop than modern economies. More recent efforts focused on why post-communist economies did not catch up to first-world economies as quickly as expected. Today, in the aftermath of the 2008 financial crisis, there is a sober realization that even the advantages (transparency, participation, rule of law) used to explain economic development are not completely accurate descriptions of the most advanced economies (Acemoglu and Robinson, 2012). One of the goals of this paper is to propose that, instead of a clear dichotomy between "natural" and "open" systems in the language of North et al (2009), or between "extractive" and "inclusive" institutions in the language of Acemoglu and Robinson (2012), it is only in the degree of closeness to the ideals (widespread concern for public good and of universal jurisdiction of criminal and tort law) that different societies differ. As Olson (2000) put it, rights-respecting markets with low predation bring more prosperity the closer a society comes to that ideal state, and less prosperity if (when) the society allows those benefits to erode. What we propose to add to this in Section 3 below is a quantitative treatment of this idea, with the goal of proposing that a smooth transit along this continuum, from the least commerce-friendly state of nature to its most enlightened hypothetical antithesis, can give rise to punctuated series of plateaus that represent distinct stages of economic development.

3 Development of the theta-pi model

Consider a hypothetical farmer who knows he can gain from bringing his goods to market to trade with other farmers, but fears that he might be molested en route or cheated in the trade. This farmer faces a classic decision situation where risk has to be taken to get a shot at a reward. To make things symmetrical, assume that the farmer also has a choice of bringing to market produce that he knows to be unsuitable for trade but which looks outwardly acceptable, or of going to market weapons or a fast horse in order to avoid paying. The farmer's decision becomes a variant of the classic prisoners' dilemma when

1. the temptation (T) to be the only cheater in a binary transaction is higher than the reward (R) of an honest transaction ($T > R$),
2. being the only cheated party (denoted by sucker and valued at S) is less valuable (more painful) than being part of a transaction where both cheat (and are mutually punished, (value = P)), and
3. $2R > T + S$, i.e. the total benefit $2R$ to both parties from an honest transaction is greater than the sum of the cheater's gain T and the cheated party's loss S.

A rational actor would prefer to stay at home if this type of interaction ($T >= R$) is perceived to be the only type or even the predominant type on offer. If we assign a value of zero to "stay at home", then S and P, being less attractive than staying at home, would be negative, and R and T would be positive.

Transactions with $T < R$ are "self-enforcing". They can happen in every society, but they predominate in societies that do not have institutional mechanisms to deal with predatory behavior. As established in section 1, this mechanism can take one of two forms.

3.1 Theta-Institutions

The first component of the proposed theta-pi model is the theta institution, which works by increasing trust as a rational consequence of an *expanded locus of utility.*

If every actor a in a population comes to expect that any counter-party b with whom a transacts will consider some fixed proportion θ of the good or ill that befalls a, as part of the whole utility that b will maximize, then we have functionally defined an institution. This institution reduces fear of predation and hence increases economic activity

Quantitatively, the variable θ between 0 and 1 enters into the valuation of every transaction. The decision maker subject to the institution rationally acts to maximize the transaction's utility to the decision maker U, plus θC, a proportion of the transaction's utility to the counter-party. In this paper, we consider a constant θ that represents a functionally neutral average for the whole affected population. We leave for future work the task of treating θ as a random variable with a certain distribution.

3.2 Pi-Institutions

The other reason for people to trust a trade partner is that the trade partner might have a reasonable fear of being caught and punished by a law enforcer. We represent this probability with the variable π and define getting caught as receiving the worst possible outcome in the pay-off matrix without any cost to the counter-party[3]. Institutionalization follows when all actors expect a random trade partner to only choose to break the law if doing so rationally follows from the values of π (and θ) perceived by the partner. Personal agreement with or acceptance of the law, and even fair application of the law, are not conditions of institutionalization (unlike the case with theta-institutions). The model works when there is universal expectation that the law will be applied in a predictable way.

For any particular economic transaction, whether solely between farmers or whether also involving craftsmen, thieves or warriors, the degree of temptation T and sucker's payoff S relative to the transaction's reward R are the main determinants of the minimal strength of θ and π institutions necessary to make

[3] As mentioned in Section 1.1.2, this is over and above the risk of the trading partner retaliating or "cheating back", because legal pursuit is only triggered after the predatory behavior has taken place, and can identify the culprit long after the transaction is over. In addition, unlike individual retaliation, the cost to the law enforcer is not a factor in the decision to pursue.

a transaction with those characteristics happen. Higher $\frac{T}{R}$ means that more are tempted to cheat, and higher $\frac{-S}{R}$ more trade is inhibited for any given probability of being cheated.

But how do the effects of θ institutions differ from the effects of π institutions? The answer requires further analysis. In section 3.3 below, we focus on economically symmetric transactions, where the values for $R, T, P\&S$ are the same for both parties. We will introduce asymmetries in θ and π later in this paper (Section 4.3.) We will defer to a separate paper economic transactions, such as employee-employer contracts, where the parties face very different payoffs for various behavior[4].

3.3 Model Description

The complete model (See Figure 1) simply consists of a decision tree that includes a choice between

1. staying home, which denotes opting out of the economy and is labeled with the short term "sit",
2. transacting in good faith, which denotes productive activity and is given the short label "make", and
3. transacting with intent to cheat or steal or engage in other activities that can be classified as predatory behavior, labeled "take".

The probability of the counter-party choosing to "make" is given by the variable E. E is constant when the population is at (competitive) equilibrium. Being cheated given a decision to "make", and being punished by the counter-party given a decision to "take", are thus both assigned probability $1 - E$. Getting caught is only a possibility in the "take" branch. So far, we are still within the realm of the traditional Prisoners' Dilemma, except for the different terminology.

The payoff considered by the decision maker is $U + \theta C$ as explained above. The strength of two types of institutions are represented by θ for the empathic institutions and by $\pi = \mathrm{P}[\text{getting caught} \parallel \text{"take" decision}]$ for the retaliatory institutions. The probability E of the counter-party choosing "make" over"take" is obtained by finding the equilibrium state after each party learns how others will behave, and excluding all "sit" parties since they will not be in the population that any trader will face. The number of interactions that participants do not sit out increases as θ and π increase. The model allows us to calculate the increase in wealth that comes from strengthening each type of institution. Hence, we can put a price on each, namely the amount that society as a whole would be willing to pay for the maintainers of each type of institution under different circumstances[5].

[4] Existing economic theory, such as supply-demand curves under monopoly or competition, already yields well known conclusions regarding the causes and effects of asymmetry in transactions.

[5] An organization entrusted with maintaining an institution of either variety may end up extracting more value than it adds. This imbalance is suggested by other researchers

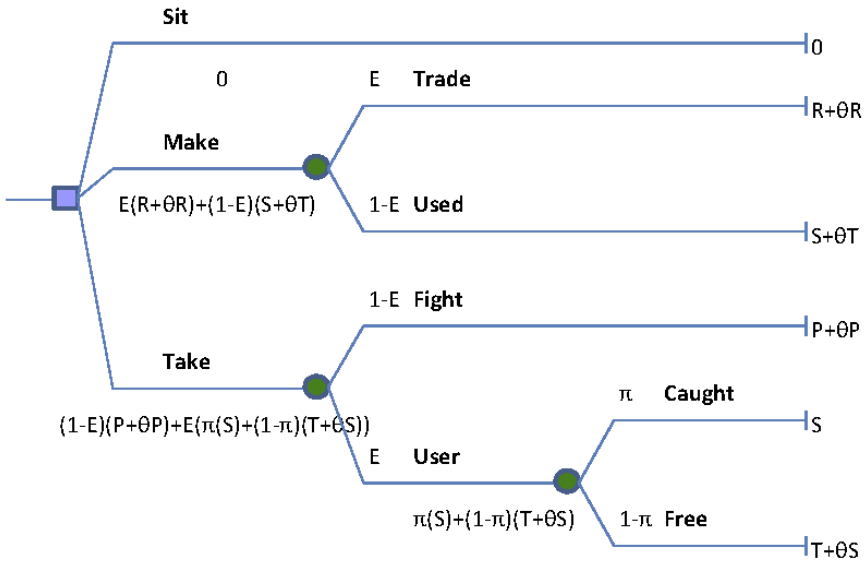


Fig. 1: Economic Actor's Decision Tree under two institutions

4 Analysis and discussion of selected scenarios

4.1 Base case: Perfect Homogeneity

Consider a homogeneous population which can conduct a single type of economic transaction with a known payoff matrix. It is relatively straightforward to find the competitive (von Neumann) equilibrium from the payoff matrix. If an equilibrium exists, then each person will follow the strategy that maximizes his payoff and there will be no uncertainty regarding the counter-party's behavior (unless the payoff matrix is such that the wining strategy is a mixed strategy[6]).

The introduction of a uniformly effective police force that catches and punishes exactly a proportion π of all predators will lead to an institution where everyone faces the same decision tree. It might take time for everyone to learn the value of π and to learn that everyone else faces the same chance of being caught. But ultimately, the population reaches equilibrium, and the probability of encountering a predator can only be 0 or 1. Which one it is depends mainly on the temptation and sucker payoffs T and S, and the police effectiveness π. Clearly, and without any further calculation, the higher π

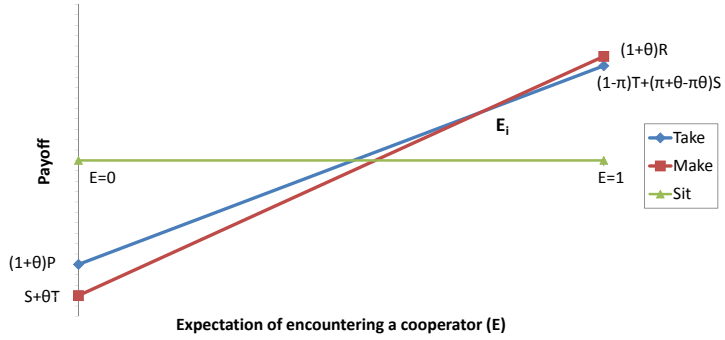
(notably Olson, 1971). We plan to analyze it in dept using the theta-pi model in a future paper.

[6] For example a game like rock-paper-scissors only has an equilibrium in mixed strategies, namely to pick one of the three move at random exactly one third of the time. The prisoners' dilemma game does not have a mixed strategy equilibrium, but some of of the derivative games used in this decision model might.

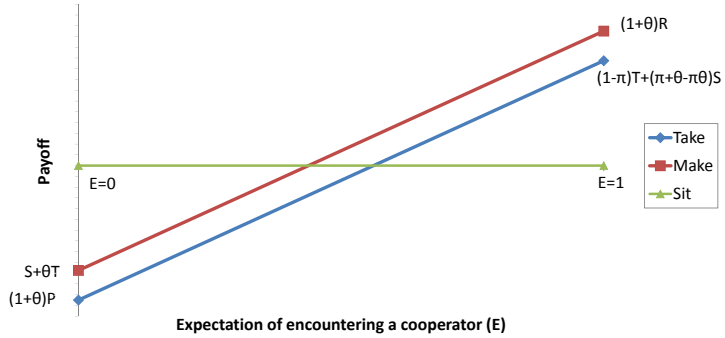
becomes, the larger the pool of transactions that will be attractive due to the addition of transactions with higher T and S.

Similarly, the introduction of a set of beliefs, be it an expectation of future benefit, a genetic kin-selection evolutionary effect, or a set of shared myths about creation, identity or destiny, can lead to a uniform level of empathy in a population. Everyone in the population feels exactly θ of the pain or pleasure of the parties with whom they interact. If everyone faces the same decision tree and preferences are identical, then institutionalized empathy will again lead to $E = 0$ or $E = 1$. As with the coercive institution, the empathic institution can expand the economy by making a transaction with higher S and T less likely to lead to a net loss than staying at home. The higher θ becomes, the larger the pool of transactions.

An algebraic exercise can help determine the relationship between the two types of institutions and the level of economic activity.



(a) "Make" and "Take" intersect



(b) "Make" always above "Take"

Fig. 2: Expected Payoff of different moves in a homogeneous population

Figure 2a represents the value of each choice in the decision tree of Figure 1, under different values of E. Remember that E is the probability of transacting

with someone who chooses the "make" move, so in a population, it is the ratio of:

1. the number of decision makers for whom "make" is the best choice, to
2. the number of decision makers who choose not to "sit" and thus put themselves in the market.

An equilibrium exists if, at $E = 1$, the value of the "make" choice exceeds the value of the "take" choice. This can occur if π and θ are such that:

$$\pi S + (1 - \pi)(T + \theta S) > R + \theta R \tag{1}$$

If this is not the case, then there is a probability of encountering a trade partner who chooses "take". In a homogeneous population, one decision maker preferring "take" means that all decision makers will prefer "take", so $E = 0$. If $E = 0$ then the value of "sit" will exceed that of the other two choices.

We mentioned in Section 1 that the equilibrium state could be path-dependent, which, in this case means that either $E = 0$ or $E = 1$ equilibrium could be reached depending on what the different parties knew or assumed about E early on. This happens when the "make" and "take" lines intersect above the "sit" line as in Figure 2a, with the value of "make" higher at $E = 1$. If we denote by E_i the value of E at this intersection, then an initial expected E to the right of E_i will lead to a "decision", and population homogeneity expands this to mean $E = 1$ at equilibrium. However, if initial E is less than E_i then the logical choice would be to "take". E approaches zero as a result, but this only means that most the population will choose to "sit". With no participants in the population, E becomes undefined

With this analysis, we can now determine the values of π and θ, at different values of T, R, S and P, that leads to different equilibrium states. Figure 3 maps the regions corresponding to different encountered equilibria along the spectrum of institutional reach:

- The region under the curve corresponds to all decision makers choosing "sit", and consequently E will decrease to zero.
- The region above the curve corresponds to the population choosing "make", and consequently E will converge to 1.
- The dashed area illustrates the region in which the steady state is sensitive to the initial expectation of market participants

The dashed area exists for certain values of T, R, S and P when not only Equation 1 is true ("make" beats "take" at E=1) but also when the "take" and "make" lines cross above the "sit" (Value = 0) line, as shown in Figure 2a. This is true when:

$$\pi < \frac{\theta^2(S\,T - P R) + \theta(T^2 + S^2 - 2\,P R) + (S\,T - P R)}{\theta^2\,T S + \theta(T^2 + S^2 - S\,T) - (S^2 + S\,T)} \tag{2}$$

The plot illustrating the different regions (Figure 3) shows that there is a threshold beyond which any combination of empathic and coercive institutions

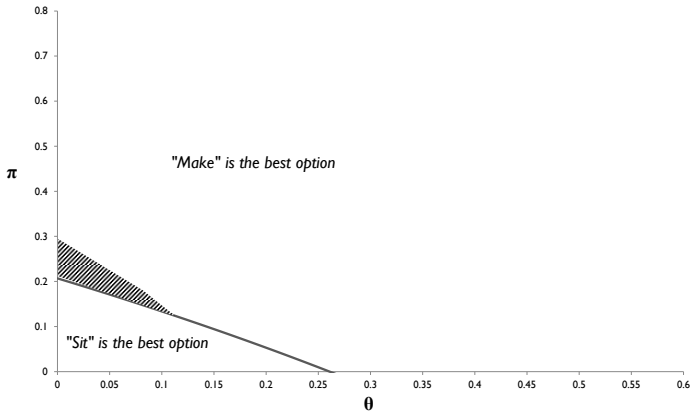


Fig. 3: Different steady state outcomes under the effect of institutions

start making if feasible for the society to benefit from any given transaction class (as typified by fixed T, R, S and P). The marginal value of an increase in the strength of each institution is the value of all economic transactions that become possible with the strengthening of that institution. This can be found by calculus provided we know how much economic opportunity is comprised of transactions with certain values of S and T to R and P. If we also know the cost of strengthening each institution, then we can formulate an optimization problem to find the least costly increase in each of π and θ needed to achieve maximum economic activity.

4.2 Variation in predominant type of transaction

To further explore the solution space of the theta-pi model for homogeneous populations, we can assume a fixed $R = -P$ value and vary the ratios of T to $-S$ to that value.

For face-to-face trade in small goods, such as at the farm fair introduced earlier, the values of $-S$ and T are in the same range as R and $-P$. The benefit of getting a bag of fruit by trickery is not much greater than the benefit of exchanging that bag for a perfectly good bag of fruit that you cannot consume. Similarly, the cost of losing a bag of fruit in an exchange against defective pottery, which might spoil or waste your oil or wine, is not much greater than the cost of exchanging bad fruit for the same leaky jug. This kind of transaction is of the low-T/low-S type, because of the relatively small difference between S and P, and between T and R.

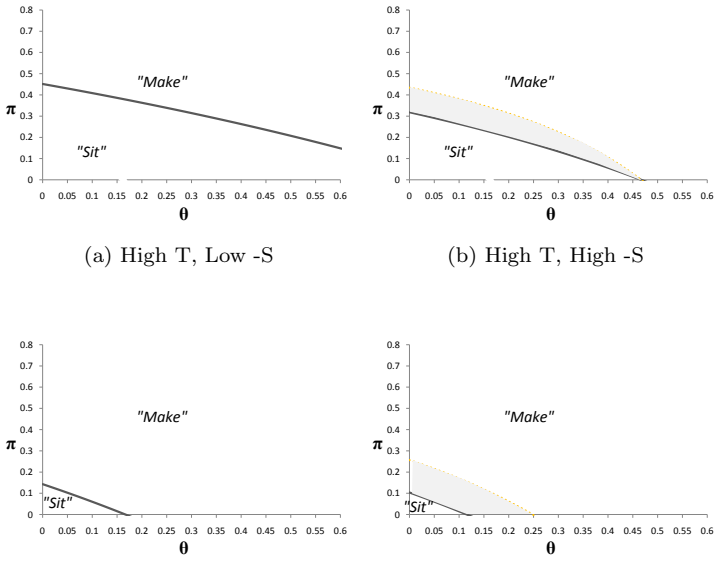
We contrast this to the case of a long-distance merchant whose trade depends on agents in far-away places. If the agent is a "taker", the merchant stands to lose not only his profit but also his capital, while the agent gains not only his cut of the profit but a whole ship-full of merchandize. Similarly,

a "taker" merchant could send an empty ship and then sue for payment for the cost of the capital as well as the lost profit. Long-distance trade offer high temptation to potential cheaters and a significant loss for the party cheated Greif (1994). In other words, we could call this a high-T/high-S trade.

For a low-T/high-S transaction, we can look at the manufacturing sector, where an assembler might procure components from different suppliers. If one component is bad, then the assembled product will lose almost all of its value, incurring a high cost for the assembler. The temptation for a single supplier to cheat on this trade is low but the sucker payoff is high.

Conversely, a wholesaler who procures goods from a big number of manufacturers and then sells them to different clients might be tempted to steal the whole load. However, each manufacturer would suffer only the loss of its own component. In this case, higher temptation is coupled with lower sucker-payoff (high-T/low-S).

Figure 4 illustrates the effect of institutions on each of the four types of transactions.



(a) High T, Low -S (b) High T, High -S

(c) Low T, Low -S (d) Low T, High -S

Fig. 4: Strength of institutions required for each category of transaction

Clearly, when temptation is low, as in the cases of the farmer and the assembler, cheaters can be completely deterred with smaller values of π and θ. The theta-pi model also tells us that both π and θ have almost equal effects on the size and the shape of the "make" region. However, the path-dependent region is broader as $-S$ increases.

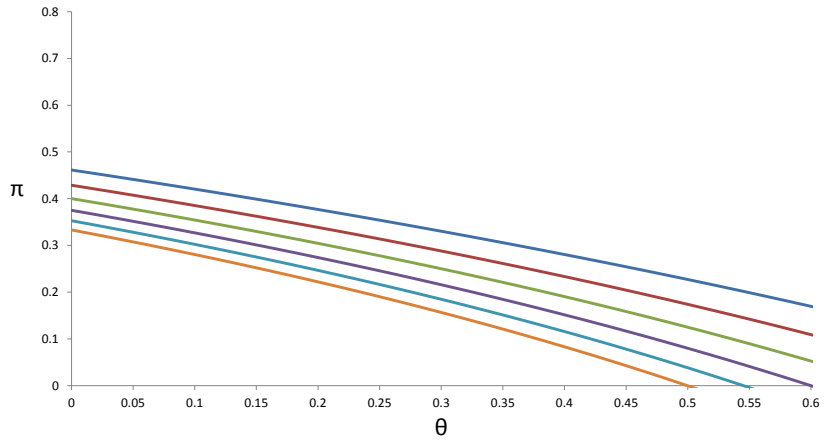


Fig. 5: Decrease in "Sit" area (T=22, R=8, P=8, S=-9,-11,. . . , -21)

When temptation is high, analyzing transaction while varying S leads to two interesting observations:

1. When sucker's payoff is low, the curve separating a healthy economy from a suppressed one is almost horizontal, meaning only π is effective. The effect of θ seems to be more important as $-S$ increases and approaches T.
2. As $-S$ increases, fewer transactions are deterred.

The first observation is mathematically substantiated by finding the straight-line approximation of the Make/Take curve and then by taking a derivative with respect to S to find the behavior of the slope as $-S$ increases:

$$\frac{T-R}{(S-T)^2} \tag{3}$$

The second observation follows from finding the exact equation of the "sit" area:

$$f := \frac{(-2SR+TR+S^2)(\ln(-2SR+TR+S^2) - ln(R-S) - ln(T-S)) - ST + SR}{S^2} \tag{4}$$

We then find the value of this expression at the limits of S:

$$\begin{aligned} \lim_{S\to -R}(f) &= \frac{(T+3R)(\ln(T+3R) - \ln(T+R) - ln(2)) + T - R}{R} \\ \lim_{S\to -T}(f) &= \frac{(T+3R)(\ln(T+3R) - \ln(T+R) - ln(2)) + T - R}{T} \end{aligned} \tag{5}$$

Because $T > R$, transactions with higher $-S$ reduce the needed strength for both π and θ to reach $E = 1$. Figure 5 illustrate how theta institutions are more effective as $-S$ increases.

4.3 Variations in reach of institution

The above analysis assumed that the institutions have identical reach and hold on the population. It is more realistic to consider a police force that can catch some perpetrators more readily than others, or an ideology of universal brotherhood or honor that resonates better with some than with others. Given enough learning time and open flow of information, rational individuals can reach an equilibrium where each not only knows his or her own π (How likely am I to get caught if I attempt predatory behavior?) and θ (How cognizant am I of the loss or gain of those with whom I transact?), but also has an accurate picture of the distribution of π and θ in the population as a whole.

Consider the case where a fraction of the population is entirely exempt from facing coercion from law enforcers. This might arise from personal characteristics (fleetness of foot, disguise skills etc.), from payoffs to corrupt law enforcers, or from ties of nepotism or employment to the enforcing organiztion[7]. The population is divided into two subgroups, each homogeneous within its ranks. If K is the percentage of the population facing a fixed value of π, then $1-K$ is the percentage of the population for whom π is equal to 0. Both face teh same *atheta*.

The converse case is when θ is zero for some sub-population (e.g. the cynical class, which may or may not be congruent with the priestly or ideological elite that serves to maintain the institution.) but π is constant for all. We will use the word "elite" in both cases to denote the exempt individuals, not by way of generalizing but simply as a convenient shorthand that satirizes the perception that those who make the rules may break the rules.

Suppose that π (or θ) is large. The non-elite will face the decision tree in which "make" is always the better option, and consequently the payoff curves will not intersect, as shown in Figure 2b. Meanwhile, the elite population, facing zero probability of getting caught (or zero empathy) could conclude from its decision tree that "take" is a better choice than "make" for all values of E.

As before, E will finally reach a ratio which is equal to the number of "makers" to the total number of agents participating in the market. E will converge to K: elite always chooses "take", while the rest "make".

However, if the elite gets too numerous, then it no longer makes any sense for anyone to "make" and the market collapse. The theta-pi model can tell us exactly the maximum elite size (or minimum non-elite size K_{min}) as a function of the market in question. There are two cases:

1. For π or θ low enough that"make" is better than "take" for the non-elite population only near $E = 1$ (see figure 2a, then $K_{min} = 1 - E_i$. This is explained by noting that when $1 - K$ is larger than E_i, E starts to

[7] It could also be said that the organization supporting the institution is collecting its dues in the form of impunity for a small class of individuals rather than in the form of direct taxation. The role of the king's retinue or "elite" in governance is much studied, (Olson, 1971; Sekeris, 2011, e.g.), but without the additional twist of empathy which we add in the theta-pi model.

converge to K, but in the process then non-elite starts to prefer "take" and the market collapses.
2. If π or θ is high enough that "make" is better than "take" for all values of E above the zero line (as in Figure 2b), then K_{min} is given by the value of E at which "make" has a value of zero for the non-elite. If K is smaller than this intersection point, the non-elite has no reason to ever leave the "sit" state[8].

4.4 Overlapping Institutions

Would universal empathic (theta) institutions, or perhaps theta-institutions that affect only those exempt from pi-institution[9], change this dynamic? Interestingly, not for long, says the theta-pi model. What happens is that the elite faces a decision tree in which "take" is only better than "make" when E is very near 1. If the elites are small enough in number, an equilibrium exists at $E = K$. If, however, the size of the elite is larger than $1 - E_i$, then, both groups will "make" until the overall expectation of encountering a cooperator (E) reaches 1. At this point, because the elites prefer the choice "take" at $E = 1$, they will once again pick that choice and start "take". As long as K does not change, the cycle starts again and keeps repeating itself. The period between each cycle depends on the rate of learning, but the oscillation continues as long as the underlying conditions remain the same.

Similar scenarios exist when the empathic institution has a limited reach and π is fixed for the overall population. A small portion of the population is then immune from the effect of empathy when engaging in market transactions.

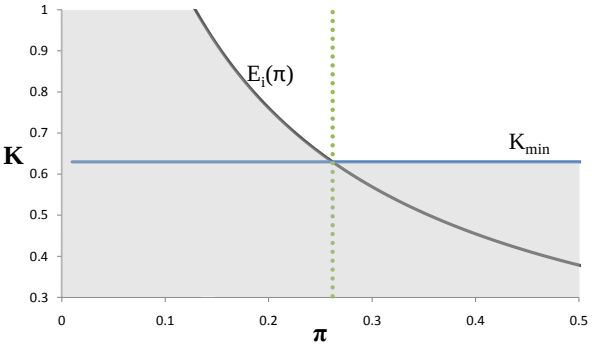
For a more generalized treatment of the case where one theta-institution overlaps with one pi-institution, we adopt the following notation. The institution that constrains the full population (elites and non-elites) will be labeled the *universal* institution. The institution that affects only the non-elites will be called the *avoidable* institution.

We distinguish three phases as the universal institution gains in strength:

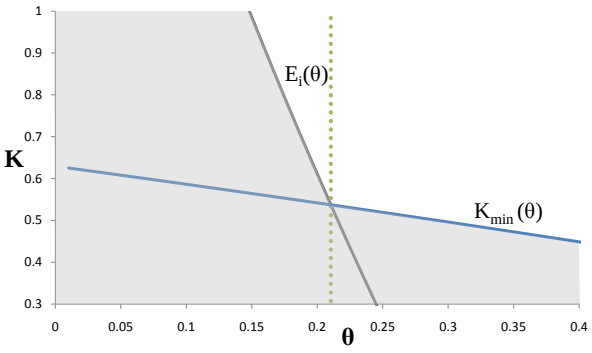
Phase 1 is when the universal institution strength, given by π or θ, is so low that the only equilibrium states are $E = 0$ and $E = K$

Phase 2 starts when the universal institution becomes sufficiently effective to lead to additional equilibrium states

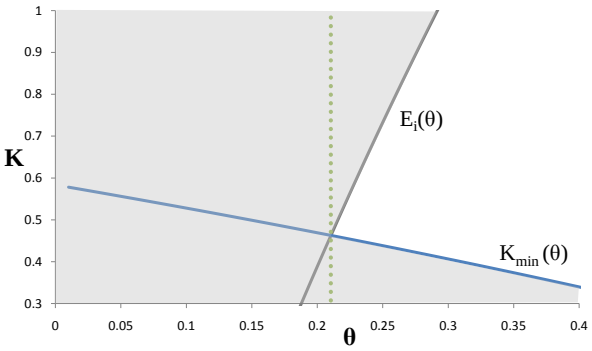
Phase 3 is when the universal institution is so strong that further increases in strength have no effect on the equilibrium state.



(a) Avoidable pi-institution, T=14, R=10, P=-10, S=-17



(b) Avoidable theta-institution, T=14, R=10, P=-10, S=-17



(c) Avoidable theta-institution, T=17, R=10, P=-10, S=-14

Fig. 6: Size of elite vs. strength of avoidable institution in Phase 1)

4.4.1 Phase 1

Phase 1 corresponds to a non-existent or weak universal institution. As the avoidable institution gains in strength, the size of the elite is constrained by $\max(E_i, K_{min})$, where:

$$K_{min} = \frac{\theta T - S}{R - S + \theta(R - T)} \tag{6}$$

And depending on the type of avoidable institution, E_i os given by:

$$E_i = \frac{P - S}{P + R - S - 2T + \pi(T - S)} \tag{7}$$

or

$$E_i = \frac{P - S + \theta(P - T)}{(1 + \theta)(P + R - T - S)} \tag{8}$$

If K is smaller than either limit, then size of the elite $(1 - K)$ is too big and the market collapses ($E = 0$). Otherwise, $E = K$. The shaded area in Figure 6 corresponds to the $E = 0$ equilibrium state while the non-shaded area corresponds to $E = K$.

Figures 6b and 6c show two different topologies for the effects of increasing θ. The transition depends on the relative values of T and S in the underlying transaction. In Figure 6c, $T > -S$, which means that high empathy might lead an actor to favor being the sucker, since there is a net gain to society from a make-take transaction[10]. Now "take" is the best option for K of the population as E approaches 1, meaning they will "take" when there are enough "makers". E_i now becomes a lower bound on the elite size $(1 - K)$, and $(1 - K_{min})$ is the only upper bound.

Figure 7 provides a visual summary of the previous analysis and the different paths to either $E = K$ or $E = 0$. Figure 8 is a sample chart portraying the changes in the elite size constraints as the universal institution increases in strength but is still below the effective threshold (i.e. still in Phase 1). The area above the curve corresponds to the equilibrium in which the elites are "taking" and the non-elites are "making" ($E = K$). The area under the curve is when both groups "take".

It is clear that until now, increasing the strength of the universal institution also allows a larger elite. E_i moves to the right, indicating lower values of the avoidable institution for the same upper bound on $(1 - K)$.

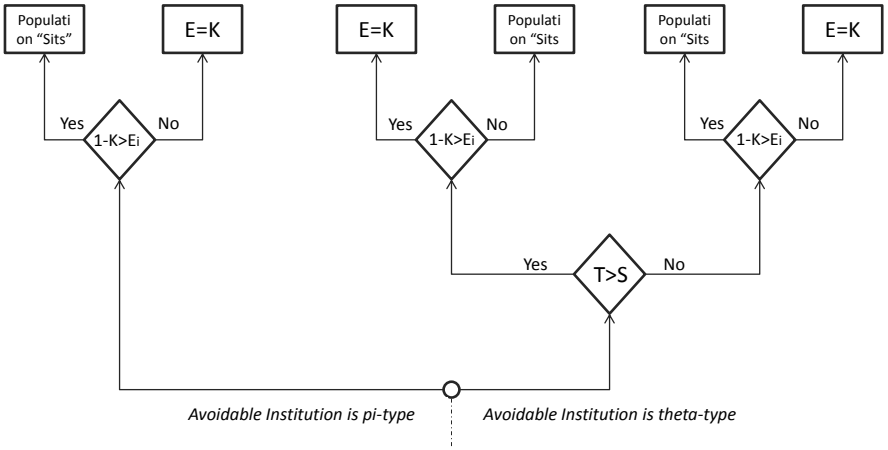


Fig. 7: Diagram of possible paths to either equilibrium state in phase 1

4.4.2 Phase 2

This phase is entered when the universal institution strength increases such that:

$$\pi_{univ} > \frac{T-R}{T-S} \tag{9}$$

or

$$\theta_{univ} > \min\left(\frac{T-R}{S-R}, \frac{-T^2-S^2+2\,PR+\sqrt{(T^2-S^2)^2-4PR(T-S)^2}}{PR-ST}\right) \tag{10}$$

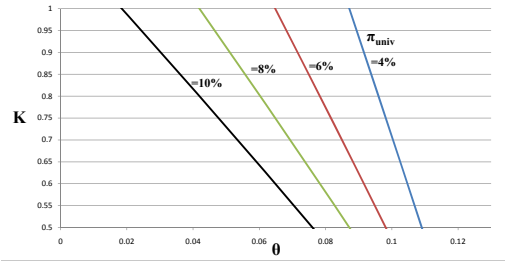
At this point, different equilibrium sates appear as well as a different set of dynamics. The elite's "make" and "take" payoff lines intersect above the zero line (as in Figure 2a), meaning they switch from "take" to "make" (or from "make" to "take") as E approaches 1. There is therefore, in addition to the original E_i for the non-elite, a new intersection point for the elite's "make" and "take" lines, which we designate as E_i^*.

Figure 9 illustrates the size constraints and the equilibrium states in this phase. Since we now have two E_i's, there are more scenarios than in Phase 1. The top horizontal line in all three sub-figures (shown in red in the color version) corresponds to E_i^* at a given universal π or θ. The hashed region

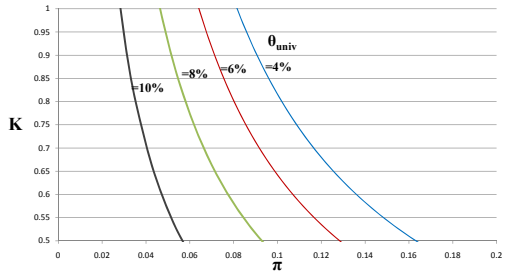
[8] Technically, for very large θ values, K might become smaller than 0.5 and the lower bound on it (K_{min}) would then be found using the make-take-sit values of the elite instead.

[9] That is, when the "noble" social stratum is also "noble" in behavior.

[10] We can call this Phase 1d for "degenerate", since transactions where being cheated is written off as a charity are unlikely to be the most prevalent in a society.



(a) Avoidable theta, Universal pi



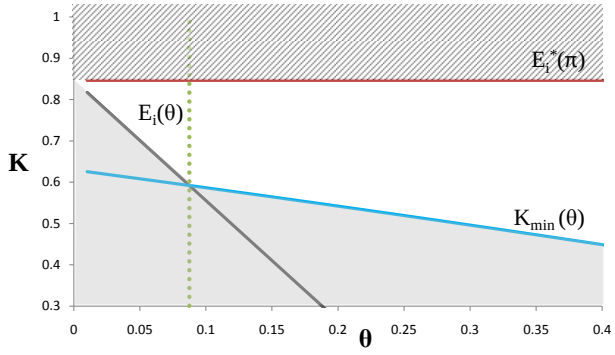
(b) Avoidable pi, Universal theta

Fig. 8: Threshold of market feasibility as universal institution strengthens (At T=13, R=10, P=-10, S=-13)

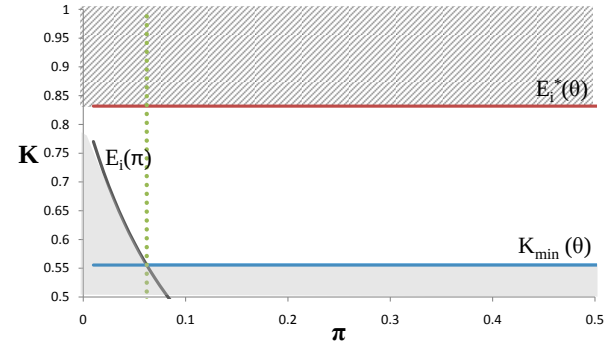
immediately above the top horizontal line in Figures 9a and 9b corresponds to $E = 1$. In Figure 9c, the dotted region immediately below the top line (shown in red in the color version) corresponds to the oscillating state described on page 16.

4.4.3 Phase 3

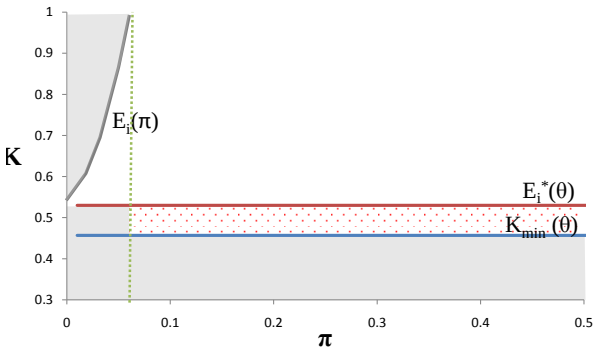
As the universal institution gets stronger, the zone in which the strength (π or θ) of the avoidable institution and its reach (K) have an effect on the outcome shrinks. In Figures 9b, 9c and 9a, the top (red) horizontal line approaches the bottom horizontal line. When the lines touch, then the universal institution can be said to be overpower the avoidable one and the equilibria as the same as in the homogeneous case (Section 4.1). The conditions for this to happen are given by Equations (11) or (12) below.



(a) Avoidable theta-institution, T=17, S=-14, $\pi_{univ} = 0.17$



(b) Avoidable pi-institution, T=14, S=-17, $\theta_{univ} = 0.17$



(c) Avoidable theta-institution, T=17, S=-14, $\theta_{univ} = 0.22$

Fig. 9: Elite size vs. avoidable institution strength (Phase 2, R=10, P=-10)

$$\pi_{univ} > \frac{TS - PR}{S(T - S)} \tag{11}$$

or

$$\theta_{univ} > \max\left(\frac{T - R}{S - R}, \frac{-T^2 - S^2 + 2\,PR + \sqrt{(T^2 - S^2)^2 - 4PR(T - S)^2}}{PR - ST}\right) \tag{12}$$

5 Findings

The addition of the theta-representation of empathy to the rational decision model interestingly shows that market creation might occur even in the absence of retributive institutions. In Section 4.2, we determined the types of symmetric transactions in which institutionalized empathy can entirely replace retributive law in deterring predation and thus permitting transactions to happen. For example, a transaction in which $T = -S = 12$, both θ and π, in the absence of one another, should be equal to 9% in order for that transaction to happen. Furthermore, stronger empathy leads to a greater variety in the types of transactions that take place.[11].

When several types of transaction exist, empathy and retribution vary in their effectiveness. Transactions where $-S$ and T are close together, are more easily fostered by theta-institutions. In the case of the farmer, whose product of trade is fully produced by one entity, 10% empathy can fully deter all predators. In the case of the long distance ship trade empathy is still highly effective but the relative effectiveness of coercive institution seem to increase. While $\theta = 20\%$ is enough to deter cheaters , $\pi = 14\%$ is equally effective. When temptation is high but the pain or loss felt by the cheated party is relatively low, as in the case of the free-loader, the effect of empathy seem to be almost non-existent, and only coercive means are effective.

When some of the population is exempt from either institution, predation is the best course for the exempt class. The development of a universal empathic institution alongside an avoidable coercive institution can lead the exempt class to change their behavior. The first difference lies in how universal empathy limits the size of the exempt population (whom we refer to with the shorthand "elite"). More elite members imply more predatory transactions, and at some point this makes everyone "sit". Universal empathy speeds up the rate at which the size of elite must decrease to maintain the market for the same transactions[12]. The second difference concerns transactions in which $T > -S$. When temptation (T) is higher than the loss ($-S$) from a particular

[11] When empathy rather than retributive action is the main institution, the "Make" line is parallel to the "Take" line, and "Make" is a better choice for a wider range of E

[12] This is explained by more a rapid increase in E_i due to both "Make"and "Take" curves moving closer together in contrast to only the "Take" curve inclining.

transaction, a significant amount of universal empathy could lead the elite to either alter its behavior in cycles, or diminish in size.

There are also some drawbacks to empathy, particularly when it is not universal. The first drawback is that there exists a zone in which some empathy is not yet effective and hence produces opposite effects: A little universal empathy in the presence of avoidable institutions can increase the tolerance of the non-elite for more predation by the elite, or to an increase in the maximum tolerable size of the elite. [13].

The second possible drawback is seen when $T > -S$: while universal empathy can indeed help avoid market collapse by allowing the elite to fill the role of "essential" predators, there is a deep chasm in the path to an ideal $E = 1$ state. For example, for transactions in which $T = 17$ and $S = -14$, and an avoidable retributive institution, $\theta = 14\%$ suffices to avoid market failure, but $\theta = 32\%$ is necessary for maximum social payoff.

6 Conclusions and further research

The theta-pi model introduced in this paper can be used to describe and analyze economically relevant institutions. The novelty of the model lies in generalizing the effects of any institution along only two dimensions. By representing the coercive aspect of an institutions with one variable that represents probability of getting caught, and the empathic aspect with one variable that describes the extent to which the locus of utility is expanded, we are able to set up and solve rational decision problems that mimic complex phenomena. These phenomena include changes in the value of each institution as a function of technological change, discontinuities in the equilibrium state of a society as a function of institutional strength and reach, and the extent to which stewardship of an institution can provide both rent-seeking opportunities and cautionary limits on that behavior.

Even without the nuances of sociological models such as Skocpol (1979), the theta-pi model can generate enough complexity to give non-obvious answers to questions such as: Under what conditions does the institution help an economy? How much value does it add? When is the party that gets the role of maintaining the institution tempted to engage in its own predatory behavior?

Some of these questions were previously investigated in a qualitative fashion (e.g. North et al, 2009). The theta-pi model further adds to the insights of North et al (2009) by showing that increasing the size of the elite not only dissipates rent, but could lead the society to completely lose the benefit of certain classes of market transactions. In special cases (see Section 4.3) the elite is periodically forced to switch in honest transactions, giving up some of its privilege, in order to continue to enjoy the full benefits of the market. Yet other circumstances place a lower bound on the size of the elite, where a

[13] The increase in universal empathy decreases E_i for the non-elite, while the elite's payoff curves remains functionally the same

smaller elite would actually reduce the economic output of the society. Technological advances and geographical expansion lead to the emergence of new types of transactions, changing the optimal behavior of the elites. Larger societies, with their greater variety in trade types, require different institutional frameworks from simpler societies.

6.1 Limitations of the Model

Although the richness of the basic theta-pi model has been demonstrated, we have not enumerated all institutional phenomena explainable by the model's basic assumptions (i.e. locus of utility expansion as a stand-in for empathic institutions and expectation of retribution by an immune party as a stand-in for coercive institutions.) We analyzed a few simple cases, but the full import of the model lies in its ability to describe complex systems of institutions with overlapping reach and continuously variable strength. Tribes and kingdoms throughout history have had webs of institutions to maintain their existence and growth, especially in light of the tendency of organizations to hijack institutions and collect rents from their control. Different overlapping jurisdictions of potentates that apply retributive justice, each with its own partially or wholly exempt class, coexist with different ethnic, religious and clan-based groupings that engender different degrees of empathy tin their in-group.

Frameworks for simplifying the representation of such institutional webs within the theta-pi model need to be developed if observed parameters are to be used to make any useful predictions in real life. However, this does not preclude elucidation of general trends, much in the same way that the multiplicity of overlapping consumer choices does not blunt the power of supply-demand equilibrium under perfect competition or monopoly.

6.2 Future Research

In future research, we need to seek ways to apply the theta-pi model to increasingly complex relationships, such as:

- multiple exempt classes with different values of the institutional parameter, e.g. $K_1, K_2 \dots K_n$ subject to $\pi_1, \pi_2, \dots \pi_n$, or continuous distributions like $\Pi(k)$, where k is the proportion of population for which $\pi \leq \Pi(k)$.
- Interactions between two overlapping institutions of the same type.
- Economic transactions that are not symmetric or not dyadic.

It might become necessary to turn to numeric solutions instead of closed-form algebraic equations when considering some of the above extensions, or perhaps employ new simplifying assumptions. But once we can mode increasingly complex general forms, the theta-pi model can be brought to bear on specific states of human social and economic organization. Circumstances of particular interest might include:

- The rise of early agrarian civilizations, following accommodations between mutually recognized theologies/priesthoods and mutually non-aggressive codes of justice/military units.
- The rise of unified coercive hierarchies under empires and unified empathic hierarchies under monotheistic religions, and their subsequent fragmentation.
- The role of company spirit and clique loyalty in the growth of a large professionally-managed business enterprize.
- Repercussions of increased flow of information on institutional equilibria in the 21st century.

The theta-pi model allows prediction of state transitions between equilibria in social-institutional systems. It will require original research to tackle the combinatorial consequences of representing sufficient contributing factors, but the theta-pi assumptions should prove a good beginning for this simplification process. The long-term rationale for pursuing this line of research is ultimately to gain understanding and insight that can inform to real-world planning of governance, economic policy and individual initiative.

References

Acemoglu D, Robinson J (2012) Why Nations Fail: The Origins of Power, Prosperity, and Poverty. Crown Business

Aktipis CA (2004) Know when to walk away: contingent movement and the evolution of cooperation. Journal of Theoretical Biology 231(2):249–260

Axelrod R (1980) More effective choice in the prisoner's dilemma. The journal of conflict resolution 24(3):379–403

Axelrod R (1987) The Evolution of Strategies in the Iterated Prisoner's Dilemma, Morgan Kaufman, Los Altos, CA, chap 2

Becker GS (1968) Crime and punishment: An economic approach. Journal of Political Economy 76(2):169–217, URL http://www.jstor.org/stable/1830482

Bolton GE, Ockenfels A (2000) ERC: a theory of equity, reciprocity, and competition. American economic review p 166193, URL http://www.jstor.org/stable/10.2307/117286

Bueno De Mesquita B, Morrow JD, Siverson RM, Smith A (1999) An institutional explanation of the democratic peace. American Political Science Review 93(4):791–807

Chong A, Calderón C (2000) Institutional quality and poverty measures in a cross-section of countries. Economics of Governance 1(2):123–135, DOI 10.1007/s101010050002

Cohen MD, Riolo RL, Axelrod R (2001) The role of social structure in the maintenance of cooperative regimes. Rationality and Society 13:5–32

Ebner A, Beck N (2008) The Institutions of the Market: Organizations, Social Systems, and Governance. Oxford University Press, USA

Greif A (1994) Cultural beliefs and the organization of society: A historical and theoretical reflection on collectivist and individualist societies. Journal of Political Economy 102(5):912–950

Greif A (2006) Institutions and the path to the modern economy: lessons from medieval trade. Cambridge University Press

Keen S (2003) Standing on the toes of pygmies: Why econophysics must be careful of the economic foundations on which it builds. Physica A: Statistical Mechanics and its Applications 324(1–2):108–116

Maynard Smith J, Price GR (1973) The logic of animal conflict. Nature 246:15–18

Neilson WS (2009) A theory of kindness, reluctance, and shame for social preferences. Games and Economic Behavior 66(1):394–403, URL http://ideas.repec.org/a/eee/gamebe/v66y2009i1p394-403.html

North D (1990) Institutions, Institutional Change and Economic Performance. Cambridge University Press, Cambridge, UK

North DC, Wallis JJ, Weingast BR (2009) Violence and Social Orders: A Conceptual Framework for Interpreting Recorded Human History, 1st edn. Cambridge University Press

Nowak M, Sigmund K (1993) A strategy of win-stay, lose-shift that outperforms tit-for-tat in the prisoner's dilemma game. Nature 364(6432):56–58

Olson M (1971) The Logic of Collective Action. Harvard University Press, Cambridge MA

Olson M (1984) The Rise and Decline of Nations: Economic Growth, Stagflation, and Social Rigidities. Yale University Press

Olson M (2000) Power And Prosperity: Outgrowing Communist And Capitalist Dictatorships. Basic Books

Olson M, Kähkönen S (2000) A Not-so-dismal Science: A Broader View of Economies and Societies. Oxford University Press, USA

Polinsky AM, Shavell S (1999a) Corruption and optimal law enforcement. Working Paper 6945, National Bureau of Economic Research, URL http://www.nber.org/papers/w6945

Polinsky AM, Shavell S (1999b) The economic theory of public enforcement of law. NBER Working Paper 6993, National Bureau of Economic Research, Inc, URL http://ideas.repec.org/p/nbr/nberwo/6993.html

Sachs JL, Mueller UG, Wilcox TP, Bull JJ (2004) The evolution of cooperation. The Quarterly Review of Biology 79(2):135–160

Scott WR (2007) Institutions and Organizations: Ideas and Interests, 3rd edn. Sage Publications, Inc

Sekeris P (2011) Endogenous elites: power structure and patron-client relationships. Economics of Governance 12(3):237–258, DOI 10.1007/s10101-010-0093-8

Selznick P (1994) The Moral Commonwealth: Social Theory and the Promise of Community. University of California Press

Sen PA (1997) Choice, Welfare and Measurement. Harvard University Press

Simon HA (1990) A mechanism for social selection and successful altruism. Science 250(4988):1665–1668, DOI 10.1126/science.2270480, URL

http://www.sciencemag.org/content/250/4988/1665
Skocpol T (1979) States and social revolutions : a comparative analysis of France, Russia, and China. Cambridge Universty Press
Von Neumann J, Morgenstern O (1967) Theory of games and economic behavior. John Wiley & Sons Inc